Exercises on Complex Numbers

Dr Iain Styles

January 9, 2009

1. If $y = x^2$, then for the following values of y, find all values of x:
   a) $y = 16$ b) $y = -25$ c) $y = -a^2$

2. State the real and imaginary parts of the following complex numbers:
   a) $z = 10$ b) $z = -5\mathrm{i}$ c) $z = 3 + 2\mathrm{i}$ d) $z = -1 + 4\mathrm{i}$ e) $z = 6 - 3\mathrm{i}$

3. Calculate the modulus r, and argument θ of:
   a) $z = 1 + \mathrm{i}$ b) $z = 2 + 2\mathrm{i}$ c) $z = -1 + \mathrm{i}$ d) $z = 3 + 4\mathrm{i}$ e) $z = -2$

4. Calculate the real and imaginary parts of:
   a) $z = \mathrm{e}^{\mathrm{i}\pi}$ b) $z = 2\mathrm{e}^{\mathrm{i}\pi/2}$ c) $z = 3\mathrm{e}^{\mathrm{i}\pi/4}$

5. Add the following pairs of complex numbers:
   a) $z_1 = 1 + 2\mathrm{i}$, $z_2 = 3 + 5\mathrm{i}$ b) $z_1 = -3 - 6\mathrm{i}$, $z_2 = 3 + 6\mathrm{i}$ c) $z_1 = \mathrm{e}^{\mathrm{i}\pi/2}$, $z_2 = \mathrm{e}^{\mathrm{i}\pi}$

6. Write down the complex conjugates of:
   a) $z = 3 + 5\mathrm{i}$ b) $z = 4$ c) $z = -2\mathrm{i}$ d) $z = 6\mathrm{e}^{\mathrm{i}\pi/3}$

7. For the following pairs of complex numbers, calculate $z_1 + z_2$, $z_1 - z_2$, $z_1 z_2$ and z_1/z_2:
   a) $z_1 = 2\mathrm{e}^{\mathrm{i}\pi/2}$, $z_2 = 3\mathrm{e}^{-\mathrm{i}\pi/2}$ b) $z_1 = \mathrm{e}^{\mathrm{i}\pi}$, $z_2 = 4\mathrm{e}^{-\mathrm{i}\pi/2}$ c) $z_1 = 1 + \mathrm{i}$, $z_2 = -1 + \mathrm{i}$