Solutions to the Exercises on Complex Numbers

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1. If $y = x^2$, then for the following values of y, find all values of x:

a) $y = 16$ b) $y = -25$ c) $y = -a^2$

Solutions:

a) $x = \pm 4$ b) $x = \pm 5\mathrm{i}$ c) $x = \pm a\mathrm{i}$

2. State the real and imaginary parts of the following complex numbers:

a) $z = 10$ b) $z = -5\mathrm{i}$ c) $z = 3 + 2\mathrm{i}$ d) $z = -1 + 4\mathrm{i}$ e) $z = 6 - 3\mathrm{i}$

Solutions:

a) $\mathrm{Re}\,(z) = 10$, $\mathrm{Im}\,(z) = 0$ b) $\mathrm{Re}\,(z) = 0$, $\mathrm{Im}\,(z) = -5$ c) $\mathrm{Re}\,(z) = 3$, $\mathrm{Im}\,(z) = 2$
d) $\mathrm{Re}\,(z) = -1$, $\mathrm{Im}\,(z) = 4$ e) $\mathrm{Re}\,(z) = 6$, $\mathrm{Im}\,(z) = -3$

3. Calculate the modulus r, and argument θ of:

a) $z = 1 + \mathrm{i}$ b) $z = 2 + 2\mathrm{i}$ c) $z = -1 + \mathrm{i}$ d) $z = 3 + 4\mathrm{i}$ e) $z = -2$

Solutions:

a) $r = \sqrt{2}$, $\theta = \pi/4$ b) $r = \sqrt{8}$, $\theta = \pi/4$ c) $r = \sqrt{2}$, $\theta = 3\pi/4$
d) $r = 5$, $\theta \approx 0.295\pi$ e) $r = 2$, $\theta = \pi$

4. Calculate the real and imaginary parts of:

a) $z = \mathrm{e}^{\mathrm{i}\pi}$ b) $z = 2\mathrm{e}^{\mathrm{i}\pi/2}$ c) $z = 3\mathrm{e}^{\mathrm{i}\pi/4}$

Solutions:

a) $\mathrm{Re}\,(z) = -1$, $\mathrm{Im}\,(z) = 0$ b) $\mathrm{Re}\,(z) = 0$, $\mathrm{Im}\,(z) = 2$ c) $\mathrm{Re}\,(z) = \frac{3}{\sqrt{2}}$, $\mathrm{Im}\,(z) = \frac{3}{\sqrt{2}}$

5. Add the following pairs of complex numbers:

a) $z_1 = 1 + 2\mathrm{i}$, $z_2 = 3 + 5\mathrm{i}$ b) $z_1 = -3 - 6\mathrm{i}$, $z_2 = 3 + 6\mathrm{i}$ c) $z_1 = \mathrm{e}^{\mathrm{i}\pi/2}$, $z_2 = \mathrm{e}^{\mathrm{i}\pi}$

Solutions:

a) $4+7\mathrm{i}$ b) 0 c) $-1+\mathrm{i} \equiv \sqrt{2}\mathrm{e}^{\mathrm{i}3\pi/4}$

6. Write down the complex conjugates of:

a) $z = 3+5\mathrm{i}$ b) $z=4$ c) $z=-2\mathrm{i}$ d) $z = 6\mathrm{e}^{\mathrm{i}\pi/3}$

Solutions:

a) $z^* = 3-5\mathrm{i}$ b) $z^* = 4$ c) $z^* = 2\mathrm{i}$ d) $z^* = 6\mathrm{e}^{-\mathrm{i}\pi/3}$

7. For the following pairs of complex numbers, calculate z_1+z_2, z_1-z_2, z_1z_2 and z_1/z_2:

a) $z_1 = 2\mathrm{e}^{\mathrm{i}\pi/2}$, $z_2 = 3\mathrm{e}^{-\mathrm{i}\pi/2}$ b) $z_1 = \mathrm{e}^{\mathrm{i}\pi}$, $z_2 = 4\mathrm{e}^{-\mathrm{i}\pi/2}$ c) $z_1 = 1+\mathrm{i}$, $z_2 = -1+\mathrm{i}$

Solutions:

a) $z_1+z_2 = -\mathrm{i}$	$z_1-z_2 = 5\mathrm{i}$	$z_1z_2 = 6$	$\frac{z_1}{z_2} = -\frac{2}{3}$
b) $z_1+z_2 = -1-4\mathrm{i}$	$z_1-z_2 = -1+4\mathrm{i}$	$z_1z_2 = 4\mathrm{i}$	$\frac{z_1}{z_2} = -\frac{1}{4}\mathrm{i}$
b) $z_1+z_2 = 2\mathrm{i}$	$z_1-z_2 = 2$	$z_1z_2 = -2$	$\frac{z_1}{z_2} = -\mathrm{i}$