Lecture 9: Quantum Teleportation

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Introduction

Last lecture, we:

- showed how a quantum computer could be used to efficiently factor large numbers

In this lecture we will:

- discuss how we can transmit quantum information

Transmitting Classical Information

- The ability to transmit classical information is something that we all take for granted
- We simply look at the information we want to send and transmit a copy of it as a sequence of digital impulses down a wire, fibre optic or radio link
- Can we do the same with quantum information?
- If our quantum system is in a state $|\psi\rangle = |010010010100110\rangle$, then we can: this state contains only classical information
- But let’s say that our system is in a state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|010010010100110\rangle + |101011010000101\rangle\right)$$

- If we know that our system is in this state, then can we transmit this in the same way?

Transmitting a Quantum State Classically

- Clearly if we know that this is our state, we can transmit it classically

Dear Bob,
My quantum system is in state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|010010010100110\rangle + |101011010000101\rangle\right)$$

Love Alice

- So all is well!
- This only works if we know what the state is.
- In general, we can’t know the state as we can’t measure the coefficients of the state vector

No-Cloning Again

- We can't send a quantum state using classical information alone, since we can't know what the state is!
- What about copying the state to a second quantum system and giving that to Bob (quantum memory stick)
- This is no good either - we can't copy the state because of the no-cloning theorem
- We *could* swap the state into a second system: $U\left|\psi\right\rangle\left|0\right\rangle = \left|0\right\rangle\left|\psi\right\rangle$
- This is allowed, but then Alice loses the state herself, AND has to send a whole quantum system to Bob
- If she does this, she might as well send the original system
- Either way, it seems that we can't just send the state: we have to send the entire quantum system

Quantum Teleportation

- The No-Cloning theorem always prevents us from making copies of a quantum state, so any method that transmits the state to Bob must lead to Alice losing the state
- The question is, do we have to send Bob the entire system, or is there a way we can just send the state?
- Quantum Teleportation does not require us to send a physical system
- QT uses the concept of entanglement to allow us to transmit a quantum state
- Let's say that Alice has a state $\left|\psi\right\rangle = a_0\left|0\right\rangle + a_1\left|1\right\rangle$ that she wishes to send to Bob
- Alice and Bob also each possess one half of an entangled pair in the state $\frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$

Preparing for Transmission

- These three qubits form a composite state

$$\begin{aligned}\left|\psi_0\right\rangle &= \left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right) \\ &= \frac{1}{\sqrt{2}}\left(a_0\left|000\right\rangle + a_0\left|011\right\rangle + a_1\left|100\right\rangle + a_1\left|111\right\rangle\right)\end{aligned}$$

- Note that the first two qubits are located physically with Alice, the third is with Bob
- Alice then performs some operations on her qubits
- First, Alice sends her two qubits through a CNOT gate giving a new state

$$\left|\psi_1\right\rangle = \frac{1}{\sqrt{2}}\left(a_0\left|000\right\rangle + a_0\left|011\right\rangle + a_1\left|110\right\rangle + a_1\left|101\right\rangle\right)$$

Preparing for Transmission

- Alice then sends the first qubit through a Hadamard gate

$$\left|\psi_2\right\rangle = \frac{1}{2}\left(a_0\left[\left|000\right\rangle + \left|100\right\rangle + \left|011\right\rangle + \left|111\right\rangle\right] + a_1\left[\left|010\right\rangle - \left|110\right\rangle + \left|001\right\rangle - \left|101\right\rangle\right]\right)$$

- Note that all measurement results on three qubits are now possible
- Alice then measures the state of her two qubits. This does something to Bob's qubit
- If Alice measures her two qubits and gets the result 00, then the state of the system after the measurement must be

$$\left|\psi'\right\rangle = a_0\left|000\right\rangle + a_1\left|001\right\rangle \equiv \left|00\right\rangle \otimes \left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right)$$

- So Bob's qubit is now in state $a_0\left|0\right\rangle + a_1\left|1\right\rangle$
- This is the state that Alice wanted to send to Bob!

Other Measurement Outcomes

- So it looks like making the measurement has teleported the original state to Bob
- But this is not the end: there were other possible results of the measurement, each of which give Bob a different state:

Alice's Result	Bob's State
00	$a_0 \|0\rangle + a_1 \|1\rangle$
01	$a_1 \|0\rangle + a_0 \|1\rangle$
10	$a_0 \|0\rangle - a_1 \|1\rangle$
11	$-a_1 \|0\rangle + a_0 \|1\rangle$

- Only one of these states is the same as the one Alice wanted to send
- But if Bob knows the result of Alice's measurement, he can apply an appropriate transformation to get the right state!

Correcting the Transmission

- What Bob must do to correct his state depends on the result of Alice's measurement
- So Alice must send the result to Bob via a classical channel
- When Bob receives the result he must apply one of the following transformations:

Alice's Result	Bob must apply
00	I
01	X
10	Z
11	ZX

- The need for Alice to send classical information to Bob stops the teleportation from being faster than light

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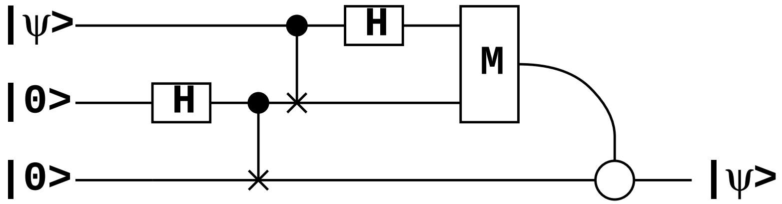
A Teleportation Circuit

A circuit which implements the teleportation algorithm is:



- In this circuit, the upper two qubits are Alice's whilst the bottom qubit is Bob's
- We include a circuit for generating the entangled pair from $|00\rangle$ on the lower two qubits
- **M** represents the act of making a measurement
- The result of the measurement feeds into some circuit that Bob uses to transform his qubit in the correct way—see the exercises

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Some Further Notes

- The state of Bob's qubit is identical to the state that Alice wanted to transmit
- But it is NOT a copy: Alice's copy was destroyed when she made the measurement
- Teleportation cannot work without the classical communications channel to transmit the result of Alices measurement
- It has actually been done!
- The state of the art is long-distance teleportation of the properties of photons of light across the River Danube (quite a simple quantum state)
- More complex quantum states have recently been teleported in the laboratory: whole atoms of Beryllium

Conclusions

In this lecture we have:

- discussed why we can’t transmit a quantum state via a classical channel
- shown how to use entanglement to facilitate the transmission of a quantum state (teleportation)
- discussed why a classical channel is also needed to make this work
- introduced a simple circuit that does teleportation

Next lecture we will:

- outline how quantum computers could lead to more secure encryption algorithms