# Quiz 1

1. Which of the following are allowed qubit states?
   (a) $|0\rangle$
   (b) $|0\rangle + |1\rangle$
   (c) $\frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$
   (d) $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$
   (e) $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$
2. The following qubit states are measured. What is $p(0)$?
   (a) $|0\rangle$
   (b) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
   (c) $|1\rangle$
   (d) $\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$
   (e) $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

3. A qubit is prepared in state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle - \sqrt{\frac{2}{3}}|1\rangle$. It is then measured. What is $p(1)$? The qubit is measured a second time. What are $p(0)$ and $p(1)$ for the second measurement, given that the first measurement gave result 1?

# Solutions

1. Which of the following are allowed qubit states?

   (a) $|0\rangle$

   *This is allowed as it is normalised*

   (b) $|0\rangle + |1\rangle$

   *This is not allowed as it is not normalised:* $|a_0|^2 + |a_1|^2 = 2$

   (c) $\frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle$

   *This is not allowed as it is not normalised:* $|a_0|^2 + |a_1|^2 = 1/2$

   (d) $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

   *This is allowed as it is normalised*

   (e) $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

   *This is allowed as it is normalised*

2. The following qubit states are measured. What is $p(0)$?

   *Use* $p(0) = |a_0|^2$

   (a) $|0\rangle$

   $p(0) = 1$

   (b) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

   $p(0) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

   (c) $|1\rangle$

   $p(0) = 0$

   (d) $\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$

   $p(0) = \left|\frac{1}{\sqrt{5}}\right|^2 = \frac{1}{5}$

   (e) $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

   $p(0) = \left|\frac{1}{2}\right|^2 = \frac{1}{4}$

3. A qubit is prepared in state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle - \sqrt{\frac{2}{3}}|1\rangle$. It is then measured. What is $p(1)$? The qubit is measured a second time. What are $p(0)$ and $p(1)$ for the second measurement, given that the first measurement gave result 1?

$p(1) = |a_1|^2 = \left|\frac{\sqrt{2}}{3}\right|^2 = \frac{2}{3}$.

*If the result of the first measurement is 1, then afterwards the state must be* $|1\rangle$

*and so for the second measurement,* $p(0) = 0$ *and* $p(1) = 1$.