Quiz 3

1. A pair of qubits is prepared in state $|\psi\rangle = \frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{6}}|10\rangle + \frac{1}{\sqrt{6}}|11\rangle$. The first qubit is measured. What is $p(0)$?

   (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$

   If the result of the measurement is 0, what is the state after the measurement?

   (a) $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$

   (b) $\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle$

   (c) $|00\rangle + |01\rangle$

2. Which of the following are entangled states?

(a) $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$

(b) $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$

(c) $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$

Solutions

1. $|\psi\rangle = \frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{6}}|10\rangle + \frac{1}{\sqrt{6}}|11\rangle$
   Measure first qubit: $p(0)$ comes from terms $|00\rangle$ and $|01\rangle$. Therefore $p(0) = \left|\frac{1}{\sqrt{3}}\right|^2 + \left|\frac{1}{\sqrt{3}}\right|^2 = \frac{2}{3}$. After the measurement (with result 0) the state is
   $$\frac{\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle}{\sqrt{\left|\frac{1}{\sqrt{3}}\right|^2 + \left|\frac{1}{\sqrt{3}}\right|^2}} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$
2. The definition of entanglement is that the state cannot be decomposed into a tensor product of individual qubit states. We therefore have
   (a) $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$. This is an entangled state as it cannot be decomposed.
   (b) $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$. This is an entangled state as it cannot be decomposed.
   (c) $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$. This is not an entangled state as it can be written as $|0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.