

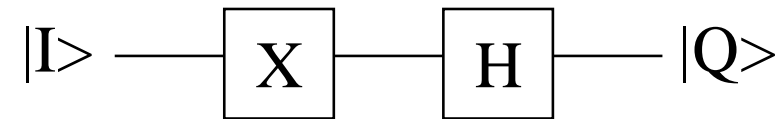
# Quiz 4

1. Consider the state  $\frac{1}{\sqrt{5}} |00\rangle + \sqrt{\frac{2}{5}} |01\rangle + \sqrt{\frac{2}{5}} |11\rangle$ . The second qubit is measured. What is  $p(1)$ ? (a)  $\frac{2}{5}$  (b)  $2\sqrt{\frac{2}{5}}$  (c)  $\frac{4}{5}$

If the result of the measurement is 1, what is the state after the measurement?

- (a)  $\sqrt{\frac{2}{5}} |01\rangle + \sqrt{\frac{2}{5}} |11\rangle$   
(b)  $\frac{2}{5} |01\rangle + \frac{2}{5} |11\rangle$   
(c)  $\frac{1}{\sqrt{2}} |01\rangle + \frac{1}{\sqrt{2}} |11\rangle$

2. Given the circuit



and the operators  $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$  and  $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ , what is the operator for the whole circuit?

(a)  $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

(b)  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

(c)  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

# Solutions

1. The state is  $\frac{1}{\sqrt{5}} |00\rangle + \sqrt{\frac{2}{5}} |01\rangle + \sqrt{\frac{2}{5}} |11\rangle$ . A measurement of the second qubit giving result 1 can arise from the terms  $|01\rangle$  and  $|11\rangle$ , so

$$p(1) = \left| \sqrt{\frac{2}{5}} \right|^2 + \left| \sqrt{\frac{2}{5}} \right|^2 = \frac{4}{5}.$$

Given that the result of the measurement was 1, the state of the system is now

$$\frac{\sqrt{\frac{2}{5}} |01\rangle + \sqrt{\frac{2}{5}} |11\rangle}{\left| \sqrt{\frac{2}{5}} \right|^2 + \left| \sqrt{\frac{2}{5}} \right|^2} = \frac{1}{\sqrt{2}} |01\rangle + \frac{1}{\sqrt{2}} |11\rangle$$

2. The order of the operators that are applied to the input state is  $X$  first, followed by  $H$ . We must therefore compute

$$HX = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$