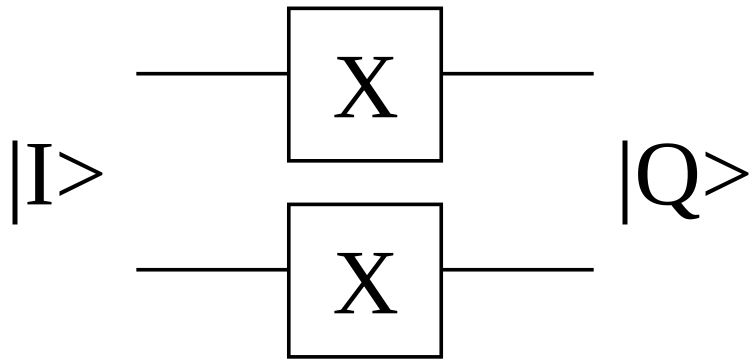
Quiz 5

1. Normalise the following states:
   (a) $|00\rangle - |01\rangle + |10\rangle - |11\rangle$
   (b) $|0\rangle - \mathrm{i}\,|1\rangle$
   (c) $2\,|0\rangle - |1\rangle$
2. Consider state $\frac{1}{2}\,|00\rangle + \frac{1}{2}\,|01\rangle + \frac{1}{\sqrt{2}}\,|10\rangle$. The second qubit is measured. What are $p(0)$ and $p(1)$? If the result is 1, what is the state after the measurement?
3. Given $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, what is the matrix for the circuit



$\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$

Solutions

1. Normalise the following states:
   (a) $|00\rangle - |01\rangle + |10\rangle - |11\rangle$
   $\sum_i |a_i|^2 = 4$, so divide by $\sqrt{4} = 2$ to get $\frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$
   (b) $|0\rangle - \mathrm{i}\,|1\rangle$ $\sum_i |a_i|^2 = 2$, so divide by $\sqrt{2}$ to get $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}\mathrm{i}\,|1\rangle$
   (c) $2\,|0\rangle - |1\rangle$ $\sum_i |a_i|^2 = 5$, so divide by $\sqrt{5}$ to get $\frac{2}{\sqrt{5}}|0\rangle - \frac{1}{\sqrt{5}}|1\rangle$
2. $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$. The second qubit is measured and we have
   $p(0) = \left|\frac{1}{2}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{3}{4}$ and $p(1) = \left|\frac{1}{2}\right|^2 = \frac{1}{4}$.
   If the result is 1, the only component of the original state that could give rise to this is $|01\rangle$, and this will be the state after the measurement.

3. To work out the matrix for the composite circuit, we need to work out the effect on the basis states. In this case it is very simple: both qubits are flipped.

$$|00\rangle \mapsto |11\rangle\,,\ |01\rangle \mapsto |10\rangle\,,\ |10\rangle \mapsto |01\rangle\,,\ |11\rangle \mapsto |00\rangle$$

We now simply transcribe the coefficients into the matrix, filling in the columns:

$$M = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$