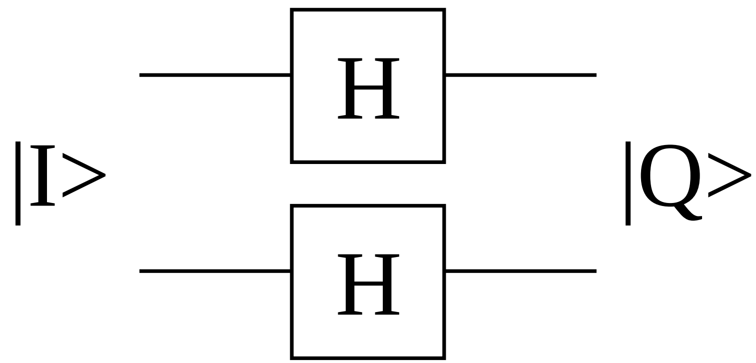
Quiz 6

1. Normalise the following states:
   (a) $\mathrm{i}\,|0\rangle - \mathrm{i}\,|1\rangle$
   (b) $\frac{1+\mathrm{i}}{\sqrt{2}}\,|00\rangle$
   (c) $|00\rangle + 2\,|01\rangle + 2\,|10\rangle$

2. Given $H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, what is the matrix for



(a) $\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$ (b) $\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$

Solutions

1. Normalise the following states:
   (a) $\mathrm{i}\,|0\rangle - \mathrm{i}\,|1\rangle$
   $\sum_i |a_i|^2 = 2$, so divide by $\sqrt{2}$ to get $\frac{1}{\sqrt{2}}\mathrm{i}\,|0\rangle - \frac{1}{\sqrt{2}}\mathrm{i}\,|1\rangle$
   (b) $\frac{1+\mathrm{i}}{\sqrt{2}}\,|00\rangle$
   Since $\left|\frac{1+\mathrm{i}}{\sqrt{2}}\right|^2 = \frac{1+\mathrm{i}}{\sqrt{2}}\frac{1-\mathrm{i}}{\sqrt{2}} = 1$, this state is already normalised.
   (c) $|00\rangle + 2\,|01\rangle + 2\,|10\rangle$
   $\sum_i |a_i|^2 = 9$, so divide by $\sqrt{9} = 3$ to get $\frac{1}{3}\,|00\rangle + \frac{2}{3}\,|01\rangle + \frac{2}{3}\,|10\rangle$

2. We first work out how the basis states transform:

$$\begin{aligned}
|00\rangle &\mapsto \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
|01\rangle &\mapsto \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \\
|10\rangle &\mapsto \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle) \\
|11\rangle &\mapsto \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)
\end{aligned}$$

We now transcribe these into the matrix, giving

$$M = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$