Introduction to Quantum and Molecular Computation

Dr Iain Styles

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The following questions are *representative* of the sorts of questions I may ask you in the exam, together with an indication of the amount of credit I would give for each question. These sample questions are drawn only from the quantum computing part of the module. The exam will be a closed-book exam, calculators will not be allowed. The exam will last for 90 minutes. The paper could contain questions drawn from any part of the module, but must cover the learning outcomes:

- Explain the differences between classical computation, quantum and molecular computation.
- Describe how to build quantum logic circuits.
- Explain various molecular and quantum algorithms.
- Outline the basic concepts of quantum mechanics.
- Describe the current practical attempts to build quantum and molecular computation devices.
- Compare the principles of quantum cryptography with classical cryptography.

There will be an element of choice in the paper, but some parts will be compulsory in order to ensure that the learning outcomes are covered.

1. **Worth 60% of the total marks.**

   (a) Discuss the key properties of quantum computers. Your answer should make reference to their potential advantages over classical computers. [15%]

   (b) For each of the following multi-qubit states, calculate the probability that a measurement of the second qubit will give results one. Write down the state of the system after the measurement

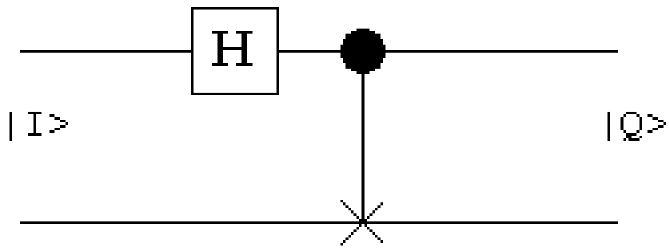
      i. $|\psi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{6}}|11\rangle$ [2%]

      ii. $|\psi\rangle = \frac{1}{\sqrt{3}}|01\rangle + \frac{\sqrt{2}}{\sqrt{3}}|11\rangle$ [2%]

      iii. $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$ [3%]

      iv. $|\psi\rangle = |1\rangle \otimes H|0\rangle$ [4%]

   (c) Compute the unitary matrix (in the computational basis) that represents the following two-qubit quantum circuit. You must show all your working. [14%]



   (d) Prove the no-cloning theorem [10%]

   (e) Describe the function of the Oracle in Grover's quantum algorithm, and explain why it is important that Grover's algorithm be terminated after a certain number of iterations. [10%]

2. **Worth 40% of the total marks**

(a) Describe how quantum computers and quantum information may affect modern cryptography. [15%]

(b) Alice wishes to exchange a private encryption key with Bob using the BB84 protocol. She generates a binary string `10101101`, and encode using `HHIHIHHI`. What is the quantum state that she transmits to Bob. [3%]

(c) Alice sends the state to Bob, who decodes using `HIHHIIHI`. They both reveal their choice of encoding. What is the string they retain? [7%]

(d) Explain how Alice and Bob can detect the presence of an eavesdropper using BB84. [15%]