Introduction to Molecular and Quantum Computation: Exercise Sheet 2

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1. Measurement of multi-qubit states

a) A system containing two qubits is prepared in the following states:

i) $|\psi\rangle = \frac{1}{3}|00\rangle + \frac{2}{3}|10\rangle - \frac{2}{3}|11\rangle$
ii) $|\psi\rangle = \frac{1}{2}|00\rangle - \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{2}|01\rangle + \frac{\mathrm{i}}{\sqrt{6}}|11\rangle$
iii) $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

The first qubit is measured. What is the probability that the result is 0? What is the probability that the result is 1? For each possible result, write down the post-measurement state, and calculate the probability that a measurement of the second qubit will give 0 and 1. Write down the states after the second measurement.

2. The No-Cloning Theorem

a) Imagine we can define a unitary operator U that can copy the qubit basis states $|0\rangle$ and $|1\rangle$:

$$U|0\rangle|0\rangle = |0\rangle|0\rangle \qquad U|1\rangle|0\rangle = |1\rangle|1\rangle$$

Can U be used to copy $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$? Verify using an explicit calculation.

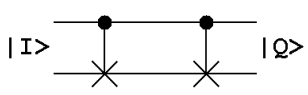
b) Imagine we can define a unitary operator U that can copy the qubit states $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$:

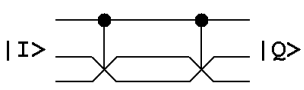
$$U|\psi_1\rangle|0\rangle = |\psi_1\rangle|\psi_1\rangle \qquad U|\psi_2\rangle|0\rangle = |\psi_2\rangle|\psi_2\rangle$$

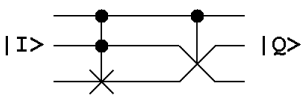
Can U be used to copy $|0\rangle$ and $|1\rangle$? Verify using an explicit calculation.

3. Practice with Quantum Logic.

For each of the following quantum circuits, find the matrix M describing the evolution of the system $|Q\rangle = M|I\rangle$. For each case, check that M is unitary.

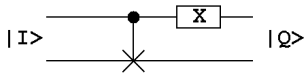
a) 

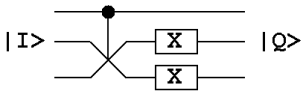
b) 

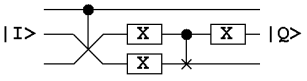
c) 

4. More Quantum Logic

For each of the following quantum circuits, find the matrix M describing the evolution of the system $|Q\rangle = M\,|I\rangle$. For each case, check that M is unitary.

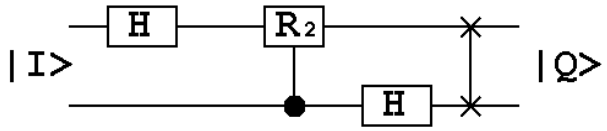
a) 

b) 

c) 

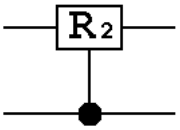
5. An Important Circuit

Find the matrix describing the following circuit:

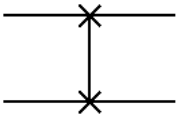


You will need the following matrices for the individual components:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$R_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathrm{i} \end{pmatrix}$$



$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$