Introduction to Molecular and Quantum Computation: Exercise Sheet 3

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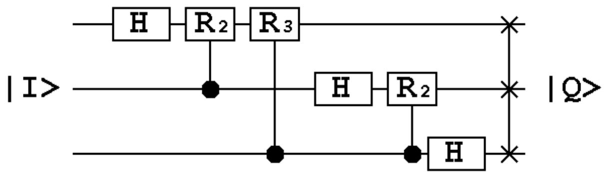
1. Three Qubit Quantum Fourier Transform

The circuit that implements the Quantum Fourier Transform on three qubits is



which is represented by the matrix

$$F_3 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 & \omega^5 & \omega^6 & \omega^7 \\ 1 & \omega^2 & \omega^4 & \omega^6 & 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega & \omega^4 & \omega^7 & \omega^2 & \omega^5 \\ 1 & \omega^4 & 1 & \omega^4 & 1 & \omega^4 & 1 & \omega^4 \\ 1 & \omega^5 & \omega^2 & \omega^7 & \omega^4 & \omega & \omega^6 & \omega^3 \\ 1 & \omega^6 & \omega^4 & \omega^2 & 1 & \omega^6 & \omega^4 & \omega^2 \\ 1 & \omega^7 & \omega^6 & \omega^5 & \omega^4 & \omega^3 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \mathrm{e}^{\pi \mathrm{i}/4}$. Show that this matrix correctly describes the three-qubit QFT by explicitly computing the QFT of $|\psi\rangle = \sum_{j=0}^{7} x_j |j\rangle$.

2. The Inverse Quantum Fourier Transform

Given the matrix for the two-qubit QFT,

$$F_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \mathrm{i} & -1 & -\mathrm{i} \\ 1 & -1 & 1 & -1 \\ 1 & -\mathrm{i} & -1 & \mathrm{i} \end{pmatrix},$$

deduce the matrix that performs the *inverse* QFT, F_2^{-1}. Can you deduce the general sum form of the inverse QFT from this matrix?