Introduction to Natural Computation

Exercise Sheet 8: Small World Networks

**Handed out:** 22/11/12.
**Due date:** 12 (noon) Wednesday 28/11/12.

This assessment is worth 2% of your overall mark for this module.

Your submission must be handed in to the School Office and you will receive a receipt.

Your work should be labelled with your student ID only. Do not include your name anywhere on the work.

You will be able to collect your work back, once it has been marked, from the School Office or in the lectures.

Marked work, with feedback, will be returned by Wednesday 5/12/12.

1. (1 mark) Give an example of small-world network which you belong to (if you cannot think of any, use your creativity!). Indicate clearly what nodes and edges in this network represent. Explain why it is a small-world network. Can you think of a member-x of the network that plays an analogous role of Kevin Bacon in the network of actors? What is your member-x number in this network? How do you compute it?

**Answer:** This answer is different for each student. A possible answer is the following. As a researcher, I publish papers in Natural Computation. I therefore belong to the network of authors of scientific papers in Natural Computation. Each node represents a researcher. There is a link in the network between two researchers if they have published a co-authored paper.

Perhaps the person with the most co-authors in the field is David Goldberg, whose role in the field is analogous to Kevin Bacon in the network of actors. I did not publish a paper with David Goldberg, so my Goldberg number is greater than one. My PhD supervisor coauthored a paper with David Goldberg. As I have a number of papers coauthored with my PhD supervisor, my Goldberg number is two, which is the least number of edges separating Goldberg from me in the network considered.

It is a small world network because:

- many authors have only a small number of co-authors, and only few authors have a large number of co-authors. The few authors with many co-authors in the network are sufficient to create short-cuts in the network that reduce substantially the length of the shortest path between any two nodes. So, even if the network is not random, *its characteristic path length is as most as short as that of a random graph*.  
- the network is not random and it presents some structural regularities: it has clusters corresponding to specific sub-areas on Natural Computation (e.g., Evolutionary Algorithms) as authors in each specific fields tend to publish more together and tend to publish less, if at all, with researchers in other sub-areas. Although not as regular as a lattice, the structure is regular enough to give the network a *clustering coefficient much better than that of a random network with the same number of nodes and edges*.

2. (1 mark) Consider the following three graphs. Which of these (if any) exhibit the small world phenomenon? Explain why / why not for each.
A: This network presents the small-world phenomenon because: (i) every two nodes are separated at most by a single node, making their distance in the graph at most two, which is considerably smaller than that of a regular lattice with the same number of nodes and edges; (ii) as the network presents a regular structure, the clustering coefficient of the network is higher than that of a random network.

B: This network presents the small-world phenomenon because: (i) from its structure it is easy to see that every two nodes are at most at distance four, which is considerably smaller than that of a regular lattice with the same number of nodes and edges; (ii) as the network presents a regular structure, a number of interlinked hubs, the clustering coefficient of the network is higher than that of a random network.

C: This network DOES NOT present the small-world phenomenon because it is a lattice-like network without any shortcuts linking distant parts of the network. Consequently, its average path length is similar to that of a completely regular lattice, and in particular, it is much larger than that of a random network with the same number of nodes and edges.