Control Theory Continued

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28 October 2010

Recap

Last week we looked at the basic of control theory. Today we're going to explore some of the details, starting with solving differential equations.

Overview

- Understand first order lag systems (which we briefly met last lecture)
- Analyse the dynamics and steady state of first order systems
- Explore PE control from them
- Touch on PI and PID control

First Order Lag Systems

- Last lecture we saw a first order lag system is any system where

$$c1\frac{dy}{dt} + c2y = x \tag{1}$$

- Here x in the input, y is the output, and $c1$ and $c2$ are some system dependent constants.
- In physical systems $c1$ represents the capacity to store energy or mass and $c2$ the resistance to change.
- Simple to analyse and control. This is what we're going to do today.

Heating a House

- Let's build a simple first order system for heating a room
- y is the output, so the temperature in this case.
- x is the input, so the energy supplied by the central heating or solar radiation.
- $c1$ the heat capacity of the room, and $c2$ how well insulated the room is.

$$c1\frac{dy}{dt} + c2y = x \tag{2}$$

Steady State Behaviour

- Given a constant input the house will eventually settle to a constant temparature.
- This is called the steady state.
- What is it? Hint: the rate of change (i.e. derivative) of temparature is zero at the steady state.

Steady State Behaviour Continued

- We have
$$c1\frac{dy}{dt} + c2y = x \tag{3}$$
- We know x is fixed and $\frac{dy}{dt}$ is zero.
- So $c2y = x$.
- Thus $y = \frac{x}{c2}$ in the steady state.

Transient Behaviour

- What about the behaviour before the steady state is reached?
- This is called the transient behaviour.
- To solve for this, we need to know how to solve simple differential equations.

Linear Differential Equations

- A first order lag system is a linear differential equation.
- We can rearrange into *standard form* and then it is straightforward to derive a solution.
- Standard form

$$k1x = y + k2\frac{dy}{dt} \tag{4}$$

Solving Standard Form

- Can read off steady state: $k1x$.
  - x is the process input
  - $k1$ is called the steady state gain
- Transient Behaviour
  - $k2$, called the time constant, tells us how quickly we reach the steady state.
  - Approximately 63% of final value reached in $k2$ units of time.

A Simple Model

- Simplied version of electric motor model we saw last week:
  - $b\frac{d\theta}{dt} = Ki$
  - $Ri = V - K\frac{d\theta}{dt}$
- Where
  - b is *back voltage*, which resists movement (equivalent to friction)
  - θ is the angle of rotation
  - K is some constant
  - i is the current
  - R is resistance
  - V is volatge

Solve It!

- Combine the two motor equations into a single equation
- Convert to standard form
- What is the steady state speed?
- What is the time constant?

Proportional Error Control

- Define *error* e as the difference between the target output y_{target} and the observed output y.
- PE control adjusts input by some factor αe
- I.e. $x_t = \alpha e_{t-1}$.

Analysing PE Control

- PE control will never reach the target value?
- Why? As we come closer to the target we decrease input.

Proportional Integral Control

- Take PE control and add in a factor for the accumulated error (the integral of error).
- $x_t = \alpha_1 e_{t-1} + \alpha_2 \int edt$
- No steady state error.
- More lag.
- More unstable.

Proportional Integral Differential Control

- Take PI control and add in a factor for the rate of change of error (the derivative of error).
- $x_t = \alpha_1 e_{t-1} + \alpha_2 \int edt + \alpha_3 \frac{de}{dt}$
- No steady state error
- Fast response to changes in error.
- More stable.
- Three parameters to tune – hard to set them all.

PID Control on the Intellbrain

- PID controller class supplied by Ridgesoft.