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(Comments and criticism welcome: Send to [a.sloman\[at\]cs.bham.ac.uk](mailto:a.sloman[at]cs.bham.ac.uk))

Notes on Mathematics, Metaphysics, Evolution The importance of chemistry for biological emergence

Expanded version of notes written after the [FraMEPhys](https://twitter.com/framephys) Levels of Explanation Workshop (<https://twitter.com/framephys?lang=en>) at the University of Birmingham 3rd April 2019.

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Background

Levels of Explanation Workshop Details here:

<https://framephys.org/framephys-workshop-april-3-2019-levels-of-explanation/>

<https://twitter.com/FraMEPhys/status/1113102348707037185>

Unfortunately I was able to attend only the last two talks, by Eleanor Knox, "From Abstraction to Explanation to Levels: some thoughts", and Alex Franklin, "How do levels emerge?".

This discussion paper attempts to summarise some of the important aspects of chemistry that are essential for the metaphysical creativity of biological evolution and which were not mentioned in the talks -- unless I missed something! This extends my previous notes on Mathematics, Metaphysics, Evolution following a FraMEPhys workshop in January 2019,

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/evo-framephys.html> (also [pdf](#)) and themes from a talk to philosophy students in Zurich in March 2019

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/zurich-talk.html>

How can a physical universe produce mathematical minds?

And why are they so hard to replicate in current AI systems?

Additional changes were made to this document in April and May 2018, aiming to clarify what I am proposing, and its implications for philosophy, science and engineering (design of intelligent machines).

(DRAFT: Liable to change)

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This paper is

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/emergent-physics.html>

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/emergent-physics.pdf>

This is an extension of the Meta-Morphogenesis project, pointing out some of the connections between products of biological evolution and varieties of dynamic metaphysical grounding.

A partial index of discussion notes in this directory is in

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/AREADME.html>

Closely related:

-- Immanuel Kant's views on mathematics (1781)

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/kant-maths.html> or [pdf](#)

-- Alan Turing (1938) on mathematical [intuition](#) vs mathematical [ingenuity](#).

He thought only the latter could be implemented on digital computers [Turing\(1938\)](#) -- discussed in:

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/turing-intuition.html> or [pdf](#)

-- The multiple roles of compositionality in biology

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/sloman-compositionality.html> (or [pdf](#))

-- Jane Austen's concept of information (Not Claude Shannon's)

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/austen-info.html> (or [pdf](#))

-- The Meta-Morphogenesis project

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.html> (Also [pdf](#))

This paper is a contribution to discussion of the chemical basis of biological emergence in a physical universe, partly inspired by Schrödinger's discussion in *What is life?* (1944) and Turing's [1952 paper](#) on chemistry-based morphogenesis.

I haven't found a short way to summarise the network of ideas, so I apologise for length, and repetition of parts of related documents. I'll welcome comments, criticisms, suggestions or corrections.

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The Chemical Basis of Emergence in a Physical Universe

Biological evolution provides strikingly creative examples of emergence which would not have been possible without chemistry, for reasons identified by Schrödinger in *What is life?* (1944).

I suggest that is essentially because quantum mechanisms (e.g. bond formation and catalysis) allow molecular structures to have [discrete](#) changes between highly stable structures alongside [continuous](#) changes such as moving together or apart, twisting and folding.

Some of that creativity includes production of new kinds of virtual machinery required for information processing by individuals and groups of individuals, about which more below. (Most of what philosophers have written about virtual machines seems to be based on ignorance of some of their important features, mentioned below.)

Those sorts of creativity involve discovery and use of (pre-existing) parametrisable mathematical structures that are instantiated in multi-layered genomes (designs for organism types) and used to generate some of their products, including foraging behaviours using chemical trails, termite cathedrals, flight control mechanisms, human languages, human mathematical discoveries and practical applications of mathematics.

Some genome instantiation is done in cooperation with the environment: in humans and other intelligent species, important genome instantiation phases are staggered (delayed while information is acquired from the environment using previously instantiated genome layers e.g. in development of linguistic, spatial manipulation and mathematical capabilities).

This blurs the boundary between machine and environment, and the results are totally different from the common view of computation as what happens in machines running fixed previously constructed programs.

Using the creativity provided by these mechanisms, evolution produced increasingly rich types of emergence, including emergence of ancient mathematical minds, and before that many animals with (unreflective) spatial intelligence.

Computer based AI systems have so far failed to replicate those forms of spatial intelligence. They cannot be replicated either using logic based theorem provers (which need to be given axioms and definitions to start from) or statistics based neurally inspired deep-learning systems, because spatial intelligence includes noticing impossibility and necessary consequences, and neither impossibility nor necessity can be derived from statistical evidence: they are not extremes of probability.

Examples

Examples include squirrels defeating "squirrel proof" bird feeders (search on youtube for examples) and this 17.5 month toddler recognizing and satisfying fairly complex 3D topological and geometrical possibilities simultaneously at her first attempt (although she had previous experience of pushing things through holes in rigid plastic or metal toys):

<http://www.cs.bham.ac.uk/research/projects/cogaff/movies/ijcai-17/small-pencil-vid.webm>

4.5 minute video -- if short of time skip till the baby appears!

(Observant parents can find many more examples!)

I have many examples that require some of the kinds of thinking that could have led to the discoveries organised by Euclid in his Elements.

Something not in Euclid, but illustrating the point: Try this:

You probably find it obvious that a planar cut can be used to slice exactly one vertex off any convex polyhedron to produce a new convex polyhedron. (What brain mechanisms could support such knowledge?)

What effect will that cut have on the total numbers of vertices, edges and faces on the

remaining polyhedron?

How do you know that no other result is possible?

(This is a simplified, disguised, special case of a well known topological theorem.)

At present, I don't think anyone knows how brains do that sort of thing. And nothing in current AI can replicate it either.

In particular, known neural mechanisms derive probabilities, and cannot find proofs of necessity or impossibility.

There are theorem provers that can derive geometrical or topological results, but only because they start from a collection of axioms expressed in a logical notation (e.g. Hilbert's axiomatisation of Euclid's Elements).

But the ancient mathematicians did not need that: rather their axioms were **products** of spatial intelligence, not arbitrarily adopted **starting points**.

(Sadly geometry is no longer taught as a standard part of education in this country -- seriously depriving young minds.)

To explain such mathematical capabilities we need to find designs for suitable virtual machines capable of running on brain mechanisms, and possibly also on future computers.

The latter may be impossible on digital computers, if some mathematical discoveries, or some forms of practical spatial reasoning, require a mixture of discrete and continuous processing: digital computers provide only the former -- they can run digital programs to **simulate** 2D and 3D continuous motion, but perceiving and understanding such motion is a different matter.

Turing on mathematical intuition vs mathematical ingenuity

I suspect this is connected with Alan Turing's unexplained remark in his thesis, that computers can emulate human mathematical **ingenuity** but not mathematical **intuition**. He didn't say why not, though I suspect it's related to his reasons for working on chemical processes over a decade later.

[I was unaware of Turing's remark till last year. Discussed at length here:
<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/turing-intuition.html> (also pdf)]

I suspect that remark is related to his (unstated) motivation for switching attention to chemistry based morphogenesis shortly before he died.

For anyone who does not already know Turing's amazing 1952 paper "The chemical basis of morphogenesis", it is nicely summarised by Philip Ball:

"Forging patterns and making waves from biology to geology: a commentary on Turing (1952) 'The chemical basis of morphogenesis'", in *Royal Society Philosophical Transactions B*

<http://dx.doi.org/10.1098/rstb.2014.0218>

Related video interview: https://www.youtube.com/watch?v=6ed54_95kP4

Returning to emergence: we need to understand how biological information processing mechanisms such as brains can produce known forms of spatial reasoning about possibility, impossibility and necessity (among many other 'spatial' processes, e.g. perceiving, controlling actions, planning future actions, explaining successes and failures, learning, acquiring new concepts, communicating etc.)

Virtual machinery running on brains

We have learnt during the last 70-80 years that whereas some information processing systems can be fully implemented in mechanical devices -- e.g. an ancient mechanical calculator able to add, subtract, multiply or divide -- more sophisticated general purpose reasoning mechanisms need virtual machines running on physical machines supporting very complex rapidly changeable information processing mechanisms.

Virtual machinery can support much richer processes than physical machinery can, including very rapid creation and changes of complex structures, e.g. syntactic and semantic structures created by your brain while you read my tortuous prose.

Most philosophers seem not to have noticed key features of that engineering progress. Some of the important points about (emergent) computer-based virtual machinery were made in this paper, criticising common philosophical accounts of functionalist philosophies of mind:

Corey Maley and Gualtiero Piccinini, Get the Latest Upgrade: Functionalism 6.3.1, *Philosophia Scientiae*, 17 (2) 2013, pp. 1--15,

https://www.researchgate.net/publication/269978551_Get_the_Latest_Upgrade_Functionalism_631

A summary, with comments on important gaps in their paper is in this messy document (in need of re-writing):

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/vm-functionalism.html#related>

One of the most important points about virtual machinery often ignored by philosophers, is that virtual machines running in a physical computer (or collection of physical computers) are typically not fully implemented in the hardware in that physical machine or network, insofar as the virtual machine's functionality depends on causal links with other things.

For the same reason your mind (one of the most sophisticated types of virtual machine that we currently know of) is not, and cannot possibly be, fully implemented in your brain, or any other unchanging bounded portion of the physical universe. For example, your mind will, in future, refer to things that did not don't yet exist, including newborn humans and new kinds of technology. It remains the same mind after those changes, but the future implementation involves both new structures in your brain and new structures and relations in the physical environment, including, perhaps, technology that has not yet been thought of.

The physical components of the implementation substrate can change over time in both biological and human-created running virtual machines. For example email systems use a very complex virtual machine composed of many different virtual machines running on hardware scattered all over the planet, and constantly changing over time, e.g. as parts are added, removed or replaced, and switched on or off at different times. Likewise animal control systems implemented in brains, nerve networks and underlying physiological mechanisms supplying energy, maintaining functionality (e.g. repairing damage) etc.

The need for such virtual machines was discovered by biological evolution millions of years before human engineers made related discoveries during the second half of the 20th century. But there are still many things we have not learnt about these amazing products of evolution (also products of their own products). So we don't know how to replicate their functionality in computer-based machines, and it may be impossible to do so if brains make essential use of mechanisms implemented in a mixture of discrete and continuous process provided by sub-neural chemistry.

Why Chemistry?

The importance of chemistry as a layer of mechanism between physics and biology seems to have been largely ignored by (analytical?) philosophers of physics and philosophical physicists -- especially the fact that chemistry-based information processing may be intrinsically richer than the types of electronics-based information processing that now support artificial computation, including AI. I once wrote (but cannot remember where) that the fact that computers are digital (i.e. based on discrete mechanisms) is not a serious limitation because a continuous process can be approximated as closely as desired by a discrete process. But that is a mistake for at least two reasons: at any choice of resolution for discrete approximation, the representation of a wildly (infinitely) fluctuating continuous process will mostly be wrong.

That's because chemical processes combine both discrete state changes (bonds forming or being released especially by catalysts) and continuous changes (folding, twisting, moving closer/further etc.)

(A few neuroscientists have begun exploring the implications regarding limitations of network models of brain processing. E.g.

Patrick C. Trettenbrein, (2016), The Demise of the Synapse As the Locus of Memory: A Looming Paradigm Shift?, *Frontiers in Systems Neuroscience* 88

<http://doi.org/10.3389/fnsys.2016.00088>)

Schrödinger noticed some of the implications of that combination of discreteness and continuity in 1944, although he was writing before AI had been born (apart from the little known speculations of Ada Lovelace a century earlier) so he said nothing directly relevant to brains or minds.

Note that discreteness can emerge from continuity (e.g. repeatedly folding a flat sheet of paper to produce multiple distinct regions) whereas continuity cannot emerge in a discrete universe.

Contrast (classical) mathematics: the infinite sequences of zeros and ones form a mathematical continuum.

I suspect some physicists would argue (mistakenly?) that the continuity in chemical processes is an illusion because fundamental physics is discrete?

[David Deutsch seems to think something like that. *The Beginning of Infinity: Explanations That Transform the World*, Allen Lane & Penguin Books, 2011, page 458.]

Particle-based physical theories may allow continuous motion, but without any "locking" to produce persistent structures that can change continuously. (E.g. work on causal sets doesn't allow that -- unless I've misunderstood it?)

Even if my description is not accurate there remains a collection of deep differences between

-- physical processes using formation and release of chemical bonds and

-- sub-chemical interacting particles, which cannot directly produce interacting partly rigid partly flexible structures like birds nests, and skeleton based mechanisms in vertebrates.

Fundamental physics is used to explain the formation of enduring structures such as galaxies and stars. But the formation of relatively rigid entities like the solid portions of planets and asteroids requires chemical bonds holding things together while acted on by strong physical forces.

A more precise formulation would also include flexible structures forming (e.g.) nets with partly fixed topology and changing geometry.

Evolution's use of (Austen/not-Shannon) information

There is an important additional point that's only implicit so far:

The main metaphysical creativity of biological evolution, including its non-reductive emergence, depends on increasingly sophisticated uses of information, not in Shannon's (syntactic) sense, but in the much older sense of "information", used for example by Jane Austen in her novels, as explained in [Jane Austen's concept of information](#).

The evolution of increasingly sophisticated kinds of (mostly virtual?) machinery producing and using information in that sense has products that resist the kinds of reduction that I think were discussed in the two talks I heard at the workshop.

E.g. such virtual machines can refer to entities, including past and future states and processes and even mythical entities, beyond the boundaries of the physical mechanisms in which the machines are implemented.

[This can be seen as a development of P.F. Strawson's claim, in *Individuals -- an essay in Descriptive Metaphysics* (1959) that some individuals have both P and M (physical and mental) properties -- though as far as I know he knew nothing about AI or computation.]

Unfortunately, in my experience, the crucial features of computer-based virtual machinery are not understood by most philosophers (even though they now use examples every day). So it's hard for them to think about the power added by chemistry-based virtual machinery.

The Maley/Piccinini account mentioned [earlier](#) is more complete than most of what I hear/or read from philosophers, but still incomplete, alas. (As far as I can tell, it has not been updated since the paper was published.)

Even Daniel Dennett gets some important things wrong (e.g. when he compares virtual machines with centres of gravity) though some of what he writes seems to agree with what I am claiming. However, he doesn't seem to have any interest in explaining how ancient mathematical discoveries were possible.

Evolution's creativity depends in part on use of multi-layered genomes where different layers are expressed at different times, and later layers are *parametrised*, i.e. they get parameters from collected consequences of earlier gene expression.

These layers control both the developing physical mechanisms in individual organisms and their developing information-processing capabilities. But the layers are products of previous evolutionary history. This idea is explained in more detail in this discussion of [meta-configured](http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-configured-genome.html) genomes:
<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-configured-genome.html>

Thus the metaphysical creativity of a particular genome can depend on the number of generations it has been involved in, and how varied the environments of previous generations have been -- including variety in the products of earlier instances of the same genome. (I.e. the kinds of cultural evolution supported by the genome.) It can also be affected by similar evolutionary processes in other species in the environment, including food, predators, competitors, and symbionts of many kinds.

As a result of all these mechanisms and processes, the increasing abilities of humans to change the planet is in part a result of how many layers of gene expression they currently have and the time gaps that allow genes expressed during earlier phases of development to gather information for later genes to make use of. This is very different from standard models of learning.

(I think it is related to Annette Karmiloff-Smith's notion of "Representational Redescription" in her 1992 book *Beyond Modularity*.)

I regard Alistair Wilson's claim "grounding is metaphysical causation" as one way of summarising the huge variety of examples of metaphysically novel products of evolutionary and developmental processes.

This depends essentially on the fact that he allows grounding to be a temporally extended process, which I think is unusual among metaphysicians.

This is "work in progress": part of an extended project with many strands, explored in a collection of online papers growing in parallel: The Meta-Morphogenesis project:

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.html>

Comments, criticisms, suggestions, questions all welcome.

Closely related documents:

Immanuel Kant's views on mathematics (1781)

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/kant-maths.html> or [pdf](#)

Alan Turing (1938) on mathematical [intuition](#) vs mathematical [ingenuity](#).

He thought only the latter could be implemented on digital computers [Turing\(1938\)](#) -- discussed in:
<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/turing-intuition.html> or [pdf](#)

[Jane Austen's concept of information](#) (Not Claude Shannon's)

The multiple roles of compositionality in biology

<http://www.cs.bham.ac.uk/research/projects/cogaff/misc/sloman-compositionality.html> (or [pdf](#))

REFERENCES AND LINKS

Copied from another document -- still to be pruned

NOTE:

A partial index of discussion notes in this directory is in

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