Alan Turing’s 1938 thoughts on mathematical reasoning

Followed by some comments and questions below.

This document is available in two formats:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/turing-quotes.html


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(I am grateful to Francesco Beccuti for drawing my attention to the fact that Turing had distinguished intuition and ingenuity.)

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By A. M. TURING, 1938

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Section 11 of
SYSTEMS OF LOGIC BASED ON ORDINALS
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11. The purpose of ordinal logics. (Page 106)
Mathematical reasoning may be regarded rather schematically as the exercise of a combination of
two faculties[*], which we may call intuition and ingenuity. The activity of the intuition consists in
making spontaneous judgments which are not the result of conscious trains of reasoning. These
judgments are often but by no means invariably correct (leaving aside the the question what is
meant by "correct"). Often it is possible to find some other way of verifying the correctness of an
intuitive judgment. We may, for instance, judge that all positive integers are uniquely factorizable
into primes; a detailed mathematical argument leads to the same result. This argument will also
involve intuitive judgments, but they will be less open to criticism than the original judgment about
factorization. I shall not attempt to explain this idea of "intuition" any more explicitly.

The exercise of ingenuity in mathematics consists in aiding the intuition through suitable
arrangements of propositions, and perhaps geometrical figures or drawings. It is intended that
when these are really well arranged the validity of the intuitive steps which are required cannot
seriously be doubted.

The parts played by these two faculties differ of course from occasion to occasion, and from
mathematician to mathematician. This arbitrariness can be removed by the introduction of a formal
logic. The necessity for using the intuition is then greatly reduced by setting down formal rules for
carrying out inferences which are always intuitively valid. When working with a formal logic, the
idea of ingenuity takes a more definite shape. In general a formal logic, will be framed so as to
admit a considerable variety of possible steps in any stage in a proof. Ingenuity will then determine
which steps are the more profitable for the purpose of proving a particular proposition. In
pre-Goedel times it was thought by some that it would probably be possible to carry this
programme to such a point that all the intuitive judgments of mathematics could be replaced by a
finite number of these rules. The necessity for intuition would then be entirely eliminated.

In our discussions, however, we have gone to the opposite extreme and eliminated not intuition but
ingenuity[**], and this in spite of the fact that our aim has been in much the same direction. We
have been trying to see how far it is possible to eliminate intuition, and leave only ingenuity. We do
not mind how much ingenuity is required, and therefore assume it to be available in unlimited
supply. In our metamathematical discussions[***] we actually express this assumption rather
differently.

Notes:
[*] (Turing) We are leaving out of account that most important faculty which distinguishes topics of
interest from others; in fact, we are regarding the function of the mathematician as simply to
determine the truth or falsity of propositions.

[**] (AaronSloman) I think that by claiming to have "eliminated" ingenuity here, Turing means
eliminating human ingenuity and replacing it with the operations of a computing machine. For
example, an automated Geometry theorem prover, such as the one reported in Gelernter et.al.
(1964) will start from a logical formalisation of something close to Euclid's axioms and postulates
and formally derive logically formulated versions of theorems in Euclid's Elements. However, for
the ancient mathematicians the axioms and postulates were *discoveries*, not arbitrarily chosen logical axioms. Modern logic-based geometry theorem provers use formalisms and inference rules discovered/invented only recently, which were unknown to the ancient geometers. Their thinking essentially involved spatial reasoning and use of diagrams.

[***] (AaronSloman) I would be grateful for a pointer to the relevant "metamathematical discussions" mentioned by Turing.

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**COMMENTS ON TURING’S DISTINCTION**

*By Aaron Sloman*

I think the above extract makes it clear that Turing thought of Turing machines, (and more generally digital computers), as capable of applying mathematical techniques (using ingenuity), but lacking in intuition, a capability that guides ingenuity by identifying the propositions for which it is worth using ingenuity to find formal proofs.

If I have interpreted him correctly he would describe current geometrical theorem provers that are able to derive theorems in Euclidean geometry by constructing proofs based on modern, logical, formulations of Euclid's axioms and postulates, as using ingenuity but not insight. Such a theorem prover needs only to understand relationships between discrete symbols in axioms, rules and derivations. It does not need to know anything about continuous spaces and continuous curves.

In contrast, for the ancient mathematicians Euclid’s axioms were not arbitrarily chosen starting points, but important mathematical discoveries based on insight, especially insight into interactions between spatial structures and processes, such as the discovery that the relation of spatial containment of non-self-crossing closed curves on a flat surface is transitive and antisymmetric: If curve C1 contains C2 and C2 contains C3, then C1 contains C3. Moreover C2 does not contain C1, and C3 does not contain C2.

Notice that the insight that makes this evident is quite remarkable insofar as it deals with simple (=non-self crossing) closed closed curves of infinitely many different shapes and sizes. If such a curve is produced by starting at a location in a plane surface and constantly moving through the surface with arbitrary (smooth or sudden) changes of direction until the moving tip of the curve meets the starting location, without ever crossing itself, the variety of possible shapes of the enclosed boundary is enormous. Yet you are able to think about the space within the boundary and the space outside the boundary and somehow understand (how?) that that there is no route anywhere in the the surface from a point inside the boundary to a point outside the boundary.

I suspect nobody has any idea what brain mechanisms make that kind of discovery of a necessary truth possible. It cannot simply be a generalisation learnt from many examples, because such a generalisation is not guaranteed to have no exceptions.

I had not specified that the surface was planar this generalisation would have been false. E.g. on the surface of a torus (a thick ring, or inflated tube of a car tyre) there are some simple (non-self crossing) closed curves that do not have a distinct inside and outside. For any such curve C, any two points on the surface of the torus can then be joined by a smooth continuous line that does not cross C. Thinking about that is left as an exercise for the reader. (For more on properties of curves on a torus see:
Turing also states that in some cases, when the propositions, geometrical figures or drawings are "really well arranged", the validity of the intuitive steps "cannot seriously be doubted".

I am sure this "cannot ... be doubted" description was not a comment on the intellectual weakness of the mathematicians in question (inabilities to doubt), but a comment on the mathematical relevance and power of the propositions, geometrical figures or drawings.

This may be Turing’s (obscure) way of expressing what Kant had claimed (less obscurely(?)) in his 1781, namely that certain kinds of knowledge do not fit either of David Hume’s two types of knowledge: they express neither purely logical consequences of defining relations between ideas (analytic knowledge) nor empirical contingent knowledge, that can be established only by observation and experiment. Such observation-based propositions can never be conclusively established because it is impossible to repeat the experiments in all parts of the universe.

In my DPhil Thesis Sloman(1962) written before I had heard of artificial intelligence and before I had learnt to program, I attempted defended Kant’s claim that there are kinds of knowledge that are non-empirical, non-analytic (not based solely on definitions and logic) and non-contingent (necessary) truths.

Such mathematical truths, unlike most scientific generalisations, don’t need to be tested in a wide range of possible physical conditions: they are not empirical. E.g. Pythagoras’ theorem for planar geometry, and the unique prime factorization theorem (the "Fundamental theorem of arithmetic", mentioned by Turing in the extract above) don’t need to be tested in situations of varying temperature, climate, etc. However, some of them are not true in all possible spaces: there are truths about planar surfaces that don’t hold for curved surfaces, e.g. the surface of a sphere, or the surface of a torus.

A varied collection of examples of impossibility and necessity can be accessed here:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html
and in several other web pages referenced in that one.

Some examples of necessity and impossibility depend on purely logical deductions from definitions or logical reasoning about combinations of truth values. Simple examples are given in this IJCAI 2017 talk
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/ijcai-2017-cog.html#logical-reasoning

Unlike such truth-table-based reasoning, many mathematical discoveries require spatial (geometrical and/or topological) insight, but not in the way that an empirical generalisation does. E.g. "Every human over seven foot tall can learn to play the violin" may or may not be true, but if it is true that is not a necessary mathematical truth, but a contingent proposition that could turn out to be true or false, depending on the outcomes of biological evolution, for example.

In contrast these are necessary truths:
"Every human under ten foot tall has a height in feet that is not divisible by three different prime numbers".
"In a 3D Euclidean space it is impossible for three planar surfaces to completely enclose a finite portion of the space." (Kant mentioned the simpler 2D version.) As far as I know, what goes on in
your brain when you convince yourself of the truth of such geometric impossibility claims, cannot be explained by any current theory or model in psychology, neuroscience, AI, or philosophy of mathematics.

In particular, neural mechanisms and deep learning mechanisms that use statistical information to derive probabilities cannot discover that something is impossible or necessarily true. Impossibility and necessity are not points on a scale of probabilities. I'll return to this below.

Kant recognized the problem and struggled to formulate requirements for an explanation.

I suspect that Turing had not studied Kant, but had independently made the same discovery about the nature of ancient mathematical modes of reasoning as Kant, namely that "the activity of the intuition" can sometimes lead to discoveries that are non-empirical, and include knowledge of non-trivial, non-definitional, mathematical truths. Some of those discovery processes differ from the kinds of "ingenuity" of digital computers that produce formal proofs, by searching in a symbolic space.

Is that consistent with Turing’s claim that "we have gone to the opposite extreme and eliminated not intuition but ingenuity"? I take that to mean that humans no longer need to use their ingenuity to find proofs in formal systems because in many (but not all?) cases computers can search for valid proofs much faster than humans do. But that still leaves unsolved the problem of modelling, replicating or explaining what Turing called insight.

An example of this relevance and power is the complementary roles of the original processes of discovery in geometry and topology by ancient mathematicians whose results were summarised and organised in Euclid’s Elements, and the more recent use of a logical formalisation of Euclidean geometry based on something like Hilbert’s axiomatisation, to derive new consequences Hilbert(1899).

Unfortunately the use of Euclidean diagrammatic constructions to make discoveries and check hypotheses is no longer taught in all schools to bright children. As a result many people who attempt to study mathematical cognition or to model intelligent spatial reasoning in computers lack any first hand experience of some of the oldest and deepest forms of mathematical cognition, although they make regular use of the more basic forms of spatial cognition that I believe underly those ancient mathematical abilities.

It is unlikely that any current philosopher of mathematics will accept the suggestion that the axioms and derived theorems produced by those ancient mathematicians "cannot seriously be doubted" (as suggested in the quotation from Turing above), especially in the light of the discovery that physical space is not Euclidean, and in the light of the historical examples of errors made by outstanding mathematicians (including Euler) reported in Lakatos(1976). Clearly some of Euclid’s axioms can be and have been doubted, and shown not to be required for the most general mathematical features of physical space.

However, there are many special cases where they are relevant -- not on the surface of a sphere or a torus, but definitely for a class of intuitively “flat” plane surfaces. These have something in common that can be expressed in different ways. E.g. see the non-standard presentation of Euclidean geometry in Scott(2014).
So there is something right in Kant’s claim that those ancient mathematical discoveries were unlike empirical discoveries such as Boyle’s law (relating pressure and volume of a gas at constant temperature), and were also not mere logical consequences of arbitrarily chosen definitions.

Sloman (1962) was my early attempt to defend Kant in this respect. Any re-formulation of Kant’s claim should not make it vulnerable to criticism based on physical space not being Euclidean or based on the fact that human mathematicians are not infallible.

A more detailed defence could be an analysis of a working design for a machine that can replicate those ancient discoveries, but we do not yet know how to build such a machine, and neither logic-based geometric theorem provers nor neural-net-based models are candidates, the former because they use modes of reasoning from pre-formalised axioms that were not available in advance to ancient mathematicians, and the latter because statistics-based neural reasoning mechanisms cannot discover impossibilities and necessary truths, which (as Kant noted) are features of mathematical discoveries.

Some of the problems in developing automated geometric reasoners are discussed in Matsuda and Vanlehn (2004).

It isn’t always noticed that human geometrical reasoning powers extend beyond the details presented in Euclid’s Elements. An example is the construction used in Mary Pardoe’s proof of the triangle-sum theorem (without any mention of the parallel axiom):
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/triangle-sum.html

There are also examples of important geometric discoveries that were not derivable from Euclid’s original axioms and postulates or Hilbert’s logical formalization, including the possibility of using the “neusis” construction to trisect an arbitrary angle, demonstrated and discussed in http://www.cs.bham.ac.uk/research/projects/cogaff/misc/trisect.html (also pdf).
Origami constructions also extend Euclidean geometry:

Contrast with logicist AI
The role of spatial intuition in making discoveries in geometry and topology was the the main basis for my 1971 criticism of McCarthy and Hayes (1969), although I used simpler examples in my paper.

McCarthy and Hayes presented an extension of predicate logic to include time, and representations of actions and other processes that occur at a time or endure for some time. They claimed that the resulting formalism is adequate for the purposes of an intelligent agent in three respects.
1. It is metaphysically adequate: it can express anything that is capable of being true or false.
2. It is epistemologically adequate: it can express anything that agents are capable of knowing or believing, and using in their reasoning.
3. It is heuristically adequate: it provides a mode of representation that allows efficient searches for proofs or refutations of hypotheses, in addition to being useful for reasoning about plans or decisions.
In response to McCarthy and Hayes, my 1971 paper, later expanded in Chapters 7 to 9 of \textit{Sloman(1978)}, argued that although claims (1) and (2) might be correct, the third claim was not correct in all cases, since for various kinds of spatial reasoning, the use of \textit{analogical} representations, such as diagrams, maps and pictures, was more heuristically powerful when formulating hypotheses and searching for proofs, at least in some domains, including Euclidean geometry. At that time I was unaware of Turing’s distinction between insight and ingenuity.

Despite the fact that my 1971 paper received a significant amount of attention in the AI community and was soon re-published in the AI journal and a Reidel collection \textit{Images, Perception, and Knowledge}, nobody at that time noticed the connection between my claims in 1971 and Turing’s comments in 1938, as far as I know.

\textbf{Confusions about analogical representations}

In what I labelled “analogical representations”, such as diagrams, maps and pictures, the semantics (i.e. the properties and relationships in whatever is depicted) are not determined by application of functions to arguments, as in Fregean representations, but by interpreting properties and relations in the representation as corresponding to properties and relations in the objects or states of affairs depicted. Typically, interpreting such a picture requires problem solving since the parts of pictures are locally ambiguous and how each part is to be interpreted will depend on how the other parts are interpreted. I.e. what represents what is context-sensitive, and, as shown by work on so-called ‘scene-analysis’ by David Huffman, Max Clowes, and others around that time finding consistent interpretations was a non-trivial task in general. (For example see \textbf{Barrow and Tennenbaum(1981)})

The need for such context-sensitive interpretation processes is unlike the standard interpretation of complex Fregean representations in which well-formed parts are interpreted by (recursively) applying functions to arguments (including higher order functions such as quantifiers).

In 1971 I had not encountered Turing’s distinction between intuition and ingenuity, and was unaware of it until very recently. I cannot tell whether Turing would have regarded my distinction between reasoning based on Fregean notations (e.g. logic and algebra) as identical with or closely related to his distinction between intuition and ingenuity, but the two distinctions seem to be closely related.

Many readers did not pay proper attention to what I had written and thought I was recommending use of representations that are \textit{isomorphic} with what they represent, despite the fact that I explicitly denied that and gave examples of 2D pictures or diagrams representing 3D scenes. 2D pictures cannot be isomorphic with 3D scenes. This is one of the reasons why it is often difficult to get computers to interpret 2D pictures (or movies) of 3D scenes (or processes) accurately, as demonstrated dramatically by \textit{Clowes(1973)}.

For many years I had hoped that I could work out how to use computers to model the use of analogical representations in making the deep discoveries of ancient mathematicians. It seems to me that nobody in AI has achieved that, and the currently fashionable deep learning mechanisms are not candidates because they cannot discover impossibilities and necessary connections, since they work with statistical evidence and derived probabilities, which are entirely different concepts. Impossibility and necessity are not extremes of probability.
So I have recently begun to wonder whether the combination of discontinuous and continuous changes in sub-neural chemical processes made possible by quantum mechanisms, as pointed out in Erwin Schrödinger (1944) (where he showed how such mechanisms might be necessary to explain aspects of biological reproduction, influencing the thinking of Watson and Crick about DNA) might one day be shown to explain how to expand the abilities of digital computers (ingenuity) with new forms of reasoning (insight). For some tentative early thoughts on this, see the (still incomplete) discussion in Sloman(2018b).

Is it possible that Turing’s surprising switch to research on chemical diffusion-reaction mechanisms, with their combination of continuity and discontinuity Turing(1952) was motivated by a similar interest? Perhaps we’ll never know the answer to that. But that question triggered the Meta-Morphogenesis project, proposed in 2012, currently in progress here: http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.html

This document is the latest (Oct 2018) addition to that project.

Connections with process perception
I have recently been exploring the idea that the biological origins of mechanisms for intelligent reasoning about spatial (topological and geometrical) structures and relationships may have evolved under pressure from requirements for perception of spatial processes, including both seeing moving structures and and seeing changing visible appearances caused by various kinds of motion of the the viewer, including forward and backward translations, sideways translations, and rotations of view direction or orientation.

Even in a static scene viewer motion can produce a wide variety of changes in what is perceived, including portions of surfaces becoming visible or invisible, relative visible lengths changing (e.g. objects looming larger when approached, or shrinking when moving away), and projected angles changing, e.g. if you look at a rectangular table top from another part of the room and see two of the corner angles as obtuse and two as acute.

Although James Gibson Gibson(1979) drew attention to some aspects of motion perception that are biologically useful (have positive affordances), e.g. changing optical flow patterns as a textured surface is approached from an angle, he merely scratched the surface. For example, if you walk through a typical botanical garden there will be a huge variety of shapes, colours, textures and static and changing patterns projected onto your retina, providing a vast amount of information about relative distances, sizes and shapes. I suspect neither AI researchers, nor neuroscientists nor psychologists understand much about what brains do with such information. As an exercise for researchers there are few videos demonstrating some of the phenomena using kitchen furniture and a pot plant here: http://www.cs.bham.ac.uk/research/projects/cogaff/movies/chairs

An important research challenge is to characterise precisely the kinds of information afforded by such structures and processes when they occur in visual perception, and to identify the brain mechanisms that extract that information and use it for reasoning about structures, processes and possible or impossible future changes (affordances) in the environment.

I suspect the mathematical discovery powers (insight mechanisms) of ancient geometers made essential use of these older mechanisms shared with other intelligent species, and additional newer meta-cognitive mechanisms unique to humans that provided abilities to reflect on and
reason about the uses of the older mechanisms.

Exploring and developing those ideas is one of the tasks for the Meta-Morphogenesis project. As far as I know, they have so far gone unnoticed in AI research on vision and robotics, mainly because researchers have made restrictive assumptions about the functions of biological vision systems, leaving out the important functions that extend Gibson’s ideas about perception of affordances and discovery of spatial impossibilities and necessities that can be used in intelligent action selection.

It may turn out that the precise mechanisms required cannot be implemented on digital computers but require the sort of Super-Turing alternative tentatively discussed in Sloman(2018b).

I suspect Turing would have made major contributions to these problems had he lived a few decades longer, perhaps showing how brains make essential use of sub-neural chemical structures and processes with a mixture of continuous and discrete changes, instead of only digital sensing and storage mechanisms and probabilistic reasoning mechanisms.

**Recurring Themes**

Recurring themes in all these examples are:

-- The discoveries involve spatial (topological and/or geometrical) reasoning -- they do not merely use logic to derive consequences from a collection of axioms;
-- Some of the discoveries express non-contingent facts, i.e. things that are necessarily true or false, since counter-examples can be seen to be impossible.
-- Human mathematical abilities are not infallible, as Lakatos(1976) demonstrated, using the fact that even great mathematicians can make mistakes. Some critics of Kant use the work of Lakatos as evidence. But that would be relevant only if Kant had claimed that mathematicians were infallible.
-- The discoveries that do not involve errors are not mere empirical discoveries liable to refutation by new examples, and they express non-contingent facts about necessary features or impossibilities.
-- That implies that the reasoning cannot be produced by mechanisms based on statistical evidence and probabilistic reasoning: mathematical claims about impossibility and necessity are not claims about low or high probabilities.

Sloman(1962) pointed out that the modal concepts used in this context (e.g. "impossible", "possible", "necessary") are not to be understood in terms of truth or falsity in all possible worlds. The statement that the ratios of lengths of sides of a planar triangle cannot be varied without also varying the angles is a statement about possible changes of a configuration in this world. There is no reason to believe that a child discovering that linked rings cannot be unlinked simply by moving them around in space is thinking about possible complete universes. I expect the same was true of ancient mathematicians discovering impossibilities and necessary connections. I conclude that “Possible world semantics” for modal concepts is irrelevant to the nature of mathematical necessities discovered by ancient mathematicians.

The same is likely to be true of a squirrel reasoning about possible and impossible ways to get nuts from a bird feeder, or a human toddler reasoning about possible actions on her toys.
It follows that current theories in neuroscience or AI postulating learning based on statistical evidence and probabilistic reasoning cannot explain how mathematical minds work.

I am exploring the possibility that standard digital computers cannot implement the required forms of reasoning either: perhaps mathematical brains make use of a mixture of discrete and continuously deformable forms of representation.

I suspect that a similar "hunch" may have motivated the research reported in Turing’s last major paper, two years before he died.

[EXAMPLES AND REFERENCES TO BE ADDED]

Related documents
My 1971 ideas about this were slightly expanded in Chapters 7, 8 and 9 of Sloman(1978), on varieties of representation and on vision, written when I still hoped it would be possible to implement the ancient modes of mathematical reasoning about geometry in AI systems, about which I am now doubtful.

More recently, I have assembled a wide variety of examples of spatial (geometrical and topological) reasoning in humans that are very different from the forms of reasoning produced by AI researchers in geometrical theorem provers using logic and the Cartesian (coordinate-based) representation of geometry. These are all challenges for future AI systems.  
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/triangle-sum.html
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/triangle-theorem.html
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http://www.cs.bham.ac.uk/research/projects/cogaff/misc/toddler-theorems.html
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http://www.cs.bham.ac.uk/research/projects/cogaff/misc/changing-affordances.html
and others referred to in those documents.

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James J. Gibson, 1979 *The Ecological Approach to Visual Perception*, Houghton Mifflin, Boston, MA,


Immanuel Kant's *Critique of Pure Reason* (1781) has relevant ideas and questions, but he lacked our present understanding of information processing (which is still too limited) http://archive.org/details/immanuelkantscri032379mbp

Imre Lakatos, *Proofs and Refutations*, Cambridge University Press, 1976,


Dana Scott, 2014, Geometry without points. (Video lecture, 23 June 2014,University of Edinburgh) https://www.youtube.com/watch?v=sDGnE8eja5o


http://www.cs.bham.ac.uk/research/cogaff/62-80.html#1971-02
A slightly expanded version was published as chapter 7 of Sloman 1978, available here.

http://www.cs.bham.ac.uk/research/cogaff/62-80.html#crp

A. Sloman, 2018a, A Super-Turing (Multi) Membrane Machine for Geometers Part 1
(Also for toddlers, and other intelligent animals)
PART 1: Philosophical and biological background 
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/super-turing-phil.html

A. Sloman, 2018b A Super-Turing (Multi) Membrane Machine for Geometers Part 2
(Also for toddlers, and other intelligent animals)
PART 2: Towards a specification for mechanisms
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/super-turing-geom.html

A. M. Turing, (1952), 'The Chemical Basis Of Morphogenesis', in Phil. Trans. R. Soc. London B 237, 237, pp. 37--72. (Also reprinted(with commentaries) in S. B. Cooper and J. van Leeuwen, EDs (2013)).

A useful summary of Turing's paper for non-mathematicians is: Philip Ball, 2015, Forging patterns and making waves from biology to geology: a commentary on Turing (1952) 'The chemical basis of morphogenesis', Royal Society Philosophical Transactions B, http://dx.doi.org/10.1098/rstb.2014.0218


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