Chapter Five

LOGICAL FORM AND LOGICAL TRUTH

Introduction

We are now ready to set out upon the last lap of Part Two, in which our main aim has been to explain how certain kinds of words and sentences can have the meanings they do have, and how their having these meanings helps to determine the conditions in which propositions which they are used to express are true. This explanation serves two important purposes. First of all, it provides an answer to the question: What sorts of things are propositions, the entities to which the analytic-synthetic and necessary-contingent distinctions are to be applied? (Cf. 2.A.1.) Secondly it helps to display the general connection between truth and meaning, between knowing and understanding, at least in a certain class of cases. This prepares the way for the discussion of some more restricted kinds of connection, in Part Three. (Part of that discussion will be anticipated in the present chapter.)

So far, except for a few rather vague and general remarks in chapter two, we have been concerned only with descriptive words, and have seen how semantic correlations between them and universals (observable properties and relations) can determine which particular objects they describe correctly, and which they do not describe correctly, depending on whether those objects are or are not instances of the universals referred to. This, however, is not the full story of what happens when such words are put together with other words to form sentences expressing propositions. In addition, we have to describe the
functions of the other words. Even if there were no other sorts of words, even if it were possible to make statements just by combining descriptive words, we should have to discuss the way in which the method of construction of a sentence contributed towards its having a certain meaning in cases where the meaning depends on how the words are arranged. In short, we must explain what the logical form of a proposition is, and how logical words and constructions work. This will now be done.

First of all, an attempt will be made to say what logical words and constructions are, that is, to characterize the notion of a "logical constant". Then the way in which the logical constants occurring in a statement help to determine its truth-conditions will be described. Finally a discussion of what makes a proposition a formal truth (i.e. true in virtue of its logical form) will serve as an introduction to some of the problems of Part Three.

5.A. Logic and syntax

5.A.1. In the sentences "Fido is black", "All cubes have plane faces", we seem to be able to distinguish words which refer to entities, whether particular objects or universals, and words which do not. Among the former are "Fido", "black", "cubes". "All" and "is" are in the latter class. The latter are commonly described as "logical" words, or "logical constants", and in this section I wish to discuss the rationale behind our selection of some words to be described as "logical", while others are "non-logical". What is so special about the words "all", "is", "not", "some", "and", etc.?

One answer which is sometimes given to this question
is that these logical words are governed by linguistic rules which are purely syntactical. That is to say, unlike the semantic rules which correlate "Fido" and "black" with non-linguistic entities (material objects or properties), the rules for the use of logical words merely correlate words with other expressions, never with non-linguistic entities. After all, such words can occur in statements which can be seen to be true merely by examining their structure, that is, merely by examining the way in which logical words occur in them, for they can occur in formal truths, such as "It is raining or it is not raining", i.e. in statements which are true in virtue of their logical form. It is claimed that all that is relevant to their being true is their syntax, or their verbal structure, whence it follows that the linguistic rules which give the logical words their meanings, since they permit structure to generate truth, must surely be rules which do not refer to anything other than verbal structure. That is, they must be purely syntactical rules.

In addition, it is sometimes argued that formal systems of axioms and rules of inference, such as any standard formulation of the propositional calculus, serve to define the primitive symbols occurring in the axioms and rules, and that these primitive symbols are our ordinary logical words. Since the axioms and rules of a formal system are concerned only with symbols and relations between symbols, no mention being made of anything non-linguistic, it appears that the rules which "define" logical words are purely syntactical.

Despite all this, I think it can be shown that the assertion that logical words are governed by purely syntactical rules is either false or so vague as to be
quite misleading.

5.A.2. Let us now see what is wrong with saying that the rules governing logical words are purely syntactical.

When we learn to use the truth-functional connectives, such as "or", and other logical words, we do not learn to use them only in logically true propositions, such as "It is raining or it is not raining", for they may occur also in sentences like "My book is on the table or you have moved it" and "Dawn is breaking or the moon is still shining". Now, how can the meaning of "or" contribute to the meanings of these sentences? How do we learn the principles according to which logical words work? This is something which has often worried people. For example, Pap, who was sure that logical words could not be ostensibly defined, wrote:

"The analogy between interpretation of descriptive constants and interpretation of logical constants seems to break down: in the case of descriptive constants we can, after having reached the primitives, go on to ostensive definition, since there is something 'in the world' which they designate. But what would it be like to show the semantic meaning of the primitive LOGICAL constants of a natural language, such as the English word 'or'?" ("Semantics and Necessary Truth", p.364. Cf. p.366.)

Faced with this problem, some philosophers have been driven to talk about subjective feelings of "hesitation" or "indecision" which are correlated with such logical words. Others, rightly rejecting this, have gone to the other extreme and abandoned the search for anything which can be correlated with such words, taking refuge in the thesis that logic can be reduced to syntax.

Surely the correct answer is that learning the meaning or function of a logical word involves learning
how to recognize the states of affairs in which statements using the word are true or false, just as learning the meaning of a descriptive word involves learning to recognize the states of affairs in which sentences employing it are true or false?

5.A.3. Consider the word "is", in the statement "Fido is black". In order to understand how it works one must, in effect, learn the following: "A sentence may be made up of a referring expression (i.e. an expression referring to a particular object), the word 'is' and a descriptive expression, in that sequence. In order to discover whether the statement expressed by the sentence is true or false, examine the particular object referred to and see whether it has the property (or combination of properties) correlated with the descriptive word."

This rule does not correlate the word "is" with any one entity, but it certainly is concerned with non-linguistic entities, though in a very general way. In order to discover whether statements using the word are true, it is not enough to examine the structure of the sentences expressing those statements. The same goes for contingent statements using the word "or". In order to understand its role in a sentence such as "Dawn is breaking or the moon is still shining", one must (at least implicitly) learn the following rule: "If S and S' are sentences expressing statements, then a new statement may be expressed by the sentence consisting of S followed by the word 'or' followed by S'. In order to discover whether the new statement is true or false, examine the facts (i.e. look to see how things are in the world) and see whether a state of affairs obtains in which at least one of the statements expressed by S and S' is true or
whether both are false". (An understanding of the two sentences S and S' is, of course, presupposed.)

In this latter case, as in the former, whether the statement is true or false does not depend merely on the structure of the sentence expressing it, and the rule for the use of "or" does not refer only to syntactical properties of sentences, but also to states of affairs, which are non-linguistic entities. It is concerned with how things are in the world, with the facts in virtue of which the disjuncts express true or false propositions. What, therefore, is left of the assertion that the rules for the use of logical words are purely syntactical?

5.4. We can see what has happened here. Rules for the use of logical constants are extremely general. The rule for "is" does not correlate it with any particular object or set of objects, nor with any particular property or set of properties (or set of descriptibility conditions). The rule for "or" does not correlate it with any specific state of affairs, but with all kinds of states of affairs. The rules are highly non-specific: they concern objects, but no specific kinds of objects; properties, but no specific kinds of properties; and states of affairs, but no specific kinds of states of affairs. The rules are "topic-neutral". They allow logical constants to occur in statements which are about anything at all: they are not restricted to statements concerning certain topics. The word "or" has the same function in "That table is wet or highly polished" and in "She is unhappy or unwell". Since the rules governing the use of logical words are topic-neutral, one cannot discover anything about the specific subject-matter of a
statement from the fact that such a word occurs in it, as one could if the word "table" occurred in it. This makes it seem that such words are not correlated with anything non-linguistic, that they are governed by purely syntactical rules.

5.A.5. It was argued that the rules for the use of logical constants must be purely syntactical since they had the consequence that statements like "It is raining or it is not raining" can be seen to be true without examining anything non-linguistic. But in order to know that such a sentence expresses a truth, it is not enough to see the marks of which it is made up: one must know also how they contribute to the meaning of the sentence as a whole, and this involves knowing in general when a sentence containing these words expresses a truth, including cases where the truth is contingent. It is not enough to know the visible structure of the sentence: one must know the functions of the various things which make up this structure. The function of a symbol is not a syntactical property, even when it is as general a function as that of the word "or". I shall show later on how a knowledge of these general functions may enable us to discover truth-values without empirical enquiry, in "freak" cases.

5.A.6. We can now see also that the argument in terms of definability of logical words by means of formal systems (see 5.A.1.) falls away, for, since a formal system cannot define the use of words in contingent statements, it cannot fully define the function of symbols like "or". (It cannot define the use of words in logically true statements either. See 2.D.3, and
2.D.3. note. Compare Appendix II.). Truth-tables can, of course, be used to explain the use of some logical words, but only for someone who knows what truth is, and, as already remarked, since propositions are, in general, true or false in virtue of how things are in the world, truth-table definitions do not merely correlate symbols with other symbols.

5.A.7. It may be objected that I have missed the point of the assertion that logic can be reduced to syntax. It is true that I have in fact used the very arguments employed by some people to defend the assertion. But then the word "syntax" is used in a sense which must be clearly defined if it is not to lead to confusion. For example, when Wittgenstein argued (in "Tractatus") that the truth of a logical proposition can be perceived in the symbol alone (6.126, 6.113), he did not regard a symbol as just a sign (e.g. a mark on paper or a sound), but a sign with a use, i.e. a sign standing in a "projective relation to the world". (See 3.12, 3.32, 3.321, 3.327, 3.262.) So when he talked about "logical syntax", he was not concerned only with what is now often meant by "syntax", namely something concerned only with combinatorial geometrical properties of signs. This, unfortunately, is not realised by some who think that the "Tractatus" supports their belief that logic can be reduced to syntax.

5.A.8. If I explain what it would be like for a logical constant to be defined by purely syntactical rules, this may make it clear exactly what I am denying. A "purely syntactical" rule for the use of a word, as I understand
the term, would specify that the truth or falsity of statements making use of that word could in all cases be determined entirely from a consideration of its verbal form, without considering the meanings of any of the words in it which were correlated with non-linguistic entities, and without considering how things are in the world.

The following rule introduces into the English language two new words, "plit" and "plat". Each word may occur at most once in any sentence, right at the beginning or right at the end. If "plit" occurs at the beginning, or "plat" at the end, the sentence expresses a truth. If "plit" occurs at the end, or "plat" at the beginning, the sentence expresses a false proposition. So "PLIT the sun is shining", "It is dark plat" and "PLIT it is raining plat" all expresses true propositions, and "It is raining plit" and " Plat all men are mortal" both express false propositions. In all cases this can be seen merely by examining the structures of the sentences.

We can tell whether the propositions expressed by such sentences are true or false: but what propositions are they? Do such sentences say anything? Is there any difference in meaning between "PLIT the sun is shining" and "It is dark plat"? Do these sentences say the same thing or something different? The mere fact that the words "true" and "false" are used in formulating rules for the use of "plit" and "plat" is not enough to guarantee that they have anything to do with true or false propositions. Similar remarks may be made about theorems in a formal system, considered simply as theorems in a formal system: what has being a theorem, that is a formula derivable according to fixed rules from a specified
combination of symbols, to do with truth or falsehood? What can mere syntactical properties have to do with truth?

5.A.9. This example shows that although there may be words whose "use" is governed by purely syntactical rules, our ordinary logical words, such as "not", "and", "is", "or", etc., are not like that, for, unlike "plit" and "plat", they can occur in sentences expressing contingent propositions, whose truth has to be established by observation. They can occur in formal truths or formal falsehoods, whose truth-value can be discovered without considering how things are in the world, but only because they have a more general employment can we speak of truths or propositions in such cases. We shall see later on, that such formal truths are merely degenerate cases and that empirical enquiries can be used to show that they are true, despite the fact that they are not necessary. (See 5.C.7.) Thus, if normal observations show that dawn is breaking, then this establishes the truth of "Dawn is breaking or dawn is not breaking", though it could be established otherwise. How could empirical observation even be relevant if such a statement were true in virtue of syntactical properties? (Compare: "Concepts which occur in 'necessary' propositions must also occur and have a meaning in non-necessary ones." Wittgenstein, "Remarks on the Foundations of Mathematics", part IV, 41. Compare also "Tractatus" 6.124.)

5.A.10. All this arose out of the question: "What is special about the logical words, that distinguishes them from descriptive words or proper names?" We have found
that it is not true in any easily intelligible sense that they are distinguished by being governed by purely syntactical rules.

One peculiarity which has turned up (see 5.A.4.) is that their rules are topic-neutral, that is, so general as not to mention any specific kind of subject-matter. For example, rules for the use of truth-functional connectives are concerned only with whether something or other is the case, without being concerned with what kind of thing is the case. Other logical words also exhibit this topic-neutrality. From the mere fact that the word "is" occurs in the sentence "My car is green", one cannot discover what the sentence is about, or what sort of thing it is about. One can tell only that if it states a truth then the particular object referred to in it satisfies the describability-conditions for the descriptive word occurring in it. This tells us nothing about which object it is, nor what kinds of properties are referred to by the descriptive word.

Topic-neutrality is not only displayed by logical words. For example, in the sentence "All red horses are red", the fact that the same word occurs in the second and fifth places is a topic-neutral aspect of the sentence which helps to specify its meaning. The fact that the second word is a descriptive expression rather than a referring expression is likewise a topic-neutral aspect. We may say that all these topic-neutral aspects of propositions are logical constants.

5.A.11. I shall take topic-neutrality of words or constructions as a necessary condition for being a logical
constant. But it is not sufficient. For example, there is a sense in which the word "good" is topic-neutral, but I do not wish to say that it is a logical constant in "He has a good appetite". In addition, there are words, such as "alas", which may occur in sentences without restricting their subject-matter in any way, but which will not be described as logical constants, since they are not relevant to the question of truth or falsity of the propositions expressed. As already remarked (in 2.B.9), the study of truth-conditions is only one aspect of the study of meanings.\(^1\)

5.A.12. The following, therefore, will serve as our definition of "logical constant":

The expression "logical constant" describes any word or feature or aspect of a sentence which helps to determine whether the sentence expresses a true or a false proposition, and which is topic-neutral, so that from the fact that it occurs in the sentence one cannot discover what things or kinds of things that statement is about.

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1. There are many expressions which are vaguely like logical constants, at least in so far as they are topic-neutral, but are not immediately relevant to questions of truth and falsity. Examples are "incidentally", "however", "moreover", "perhaps", "probably", "nevertheless", "it seems that ....", "of course," "obviously", etc. These all have some kind of "pragmatic" meaning. That is, they concern some relation between the speaker or hearer and what is being said (e.g. surprise, or absence of surprise, hesitancy, etc.). They are of the same general nature as the following: "As you would expect ....", "Much to my surprise ....", "It is clear to me that ....", "In my opinion, for what it's worth ....", "I am inclined to think that ....", "The evidence available to me seems to show that ....", "I confidently predict that ....", and so on. (See remark about appropriateness-conditions in 2.B.9.)
(It should be recalled that I am talking only about relatively simple statements using words which can refer to or describe material objects in virtue of their observable properties and relations. Cf. 1.C.2.)

In so far as two or more statements have logical constants in common, we can describe them as having a common "logical form". The logical form of a proposition, therefore, is determined by the topic-neutral words or constructions used to express it which help to determine the conditions in which it is true or false. (Cf. Section 5.B, and 5.B.4.)

In general, the sentences which we use to make statements or ask questions about things in the material world contain some parts or features which determine the particular things and properties talked about, and others which do not limit the subject-matter in any way, but merely help to determine the kind of thing which is being said, or the way in which it is said. The latter are the logical constants and comprise the logical form of the statements. Thus, the sentences "All books are red", "Not all books are red", and "Some books are red" say different kinds of things though the subject-matter is the same: they refer to the same things and properties. Similarly, in each of the following pairs of sentences the same kind of statement is made, but about different topics:

1. We could generalize this and show how logical constants determine whether a statement, question or command is expressed. Cf. 2.B.3.)
5.A.13. We see therefore, that the logical forms of propositions correspond to common "structures" of sentences (where the occurrence of logical words and other topic-neutral aspects counts as part of the structure.) The logical form common to a pair of propositions may be represented symbolically in the usual way, by removing all non-logical words and replacing them by symbols called "variables" which indicate the kinds of words whose places they take, and whether the same word occupies two or more places in the sentence. These symbols may be described as "sentence-matrices", such as "All P's are Q", or "x is not P". Different kinds of letters may be used to indicate positions of different kinds of non-logical words, such as capitals for descriptive words or expressions, and small letters for expressions referring to particulars. Such a sentence-matrix represents the structure common to a family of sentences obtained by replacing the variable-letters by suitable non-logical words. It therefore also represents the logical form corresponding to that structure. (The symbol is not itself the structure, any more than a blue-print is the structure of the houses whose common structure it represents. The structure is a property of statements. It is neither a symbol nor a physical object, and you will not find it left behind when the things which are not the structure are removed from a sentence, any more than the structure of a house is left behind when the bricks and other materials are removed. This point can cause confusion and lead to talk about "unsaturated" entities, which cannot exist on their own, etc.)

5.A.14. It is possible to study the logical properties
and relations of propositions by classifying them according to their logical forms (for reasons which will become evident later on). Symbols are usually used to represent those logical forms in the manner just described, and systems of symbols represent sets of propositions. In Appendix II, I shall try to show briefly how a concentration on the geometrical properties of such systems of symbolic representations can lead to muddles about logic. Even where it does not lead to muddles, this concentration on geometrical (or syntactical) properties of sentences or symbols, cannot, unless accompanied by a study of the functions of words and symbols in determining the meanings of sentences, explain anything. It can, at best, lead to description and classification of logical properties of propositions and inferences. I shall try to explain, leaving the classification to others.

5.A.15. To summarize. I have tried to show that the distinguishing feature of logical constants is not that they are governed by syntactical rules, but the fact that their rules are topic-neutral. I have not yet described in any detail the ways in which such words and constructions contribute to the meanings of sentences which include them, and this will be attempted in the following sections. I hope eventually to provide an explanation of the fact that some propositions are formal truths (i.e. true in virtue of their logical form) and the fact that some inferences are formally valid (i.e. valid in virtue of their logical form), by showing how this comes about.

Once we understand what sorts of functions logical words can have, we can see what is involved in using a
logical word with one meaning rather than another, and can apply criteria for identity of meanings of logical words. This explains how it makes sense to say that the English word "and" means the same as the German word "und", and could take us one step further in the programme of chapter two. (See 2.A.) But I shall not go into this aspect in any detail, since ambiguity of logical constants does not cause as much trouble in connection with the analytic-synthetic distinction as ambiguity of descriptive words. (See 2.C.)

5.B. **Logical techniques**

5.B.1. So far I have explained in a vague sort of way how to pick out those parts or aspects of sentences which are purely logical, namely by seeing whether they are topic-neutral. I have not yet said how they work, how their occurrence in sentences contributes to the meanings of statements which they express, but will do so now. The explanation will be extended in the next section to show how it is possible for a statement to be true in virtue of its logical form. Later on, the account will be generalized to show how it is possible for a statement to be analytic.

My descriptions of the functions of logical words will have to be greatly oversimplified, and it will not be possible to make more than a few qualifications near the end, in 5.E.

5.B.2. If, as pointed out in the previous section, it will not do to say that the functions of logical words, such as "or" and "not" are defined by the recursive rules of a formal system, then how can we explain what their
functions are? What are the topic-neutral rules which enable them to occur significantly in sentences expressing contingently true or false propositions? The answer, as suggested in 5.A.2, seems to be that the rules help to specify the conditions in which statements employing those words are true, and conditions in which they are false. Learning to use logical words and constructions in sentences involves learning general principles for recognizing conditions in which statements are true or false. The rest of the chapter will simply be an amplification of this statement.

(It is notorious that no matter how much one says about what words mean, it is always possible for the objection to be made that the account is either circular, since it presupposes what it explains, or incomplete, since it presupposes something else — one of the facts which seems to have led to the doctrine of the "unsayable" in Wittgenstein's "Tractatus. So the most that I can hope to do is draw attention to certain features of what we all know about our use of logical words, in the hope that this will remove some puzzles. When I say "we all know", I do not wish to imply that we can explicitly formulate the knowledge. See Appendix III on "Implicit Knowledge".)

5.B.3. Frege, Russell and Wittgenstein each tried at some stage to make use of the notion of a function (not in the sense of a "role", but in the sense defined below) to explain our use of logical and other words. I shall use a slightly different notion, the notion of what I call a "rogator", which will be explained presently. First I shall say what is meant by a function. (The account will be brief, as the notion is familiar.) Then I shall
show why talking about such things seems to help, and
after that I shall offer my own variation on the theme.

5.B.3.a. The notion of a "function" may be defined as
follows. A function is a rule, or a principle or a mapping
which correlates entities, called "arguments", with other
entities, called "values". More precisely, a function
correlates sets of arguments with values, one value to
each argument-set. A given function will have a
restricted domain of definition, so that not every set
of objects can be an argument-set with which the function
correlates a value. The class of argument-sets for
which a function has a value is called its "domain", or
"domain of definition". A function is defined, or set
up, by specifying a domain of definition, and by stip-
ulating either some generally applicable principle or
technique, or method of calculation, which enables the
value of the function to be discovered for every argument-
set of its domain, or simply by enumerating the arguments
and the values correlated with them. (A function "has"
or "yields" a value for a given argument-set, viz. the
one which it correlates with the set. The argument-set
"yields" a value for that function. The function may be
said to be "applied" to its arguments, or to argument-
sets, to yield its values.) Normally the value of any
function for an argument-set depends on the order of
the arguments in the set, and if we restrict ourselves
to functions whose domains contain only ordered sets with
the same number of elements, say \( n \), then we can speak of
the function as "having \( n \) argument places" or as an
"\( n \)-ary function", and can speak of an argument as occurring
"in the \( i \)-th place" in some argument set. The number of
argument-places is usually indicated in a name or sign
for the function by a string of so called "variable-
letters", one for each place. (Sometimes the letters
are called simply "variables").

5.B.3.b. If words or signs referring to arguments are
substituted for each of the variable-letters in the
sign for the function, and if the ordered set of objects
corresponding to the ordered set of argument-signs lies
in the domain of definition of the function, then the
new sign thus obtained is taken as referring to the value
of the function for that argument-set (or, more simply,
for those arguments). Thus, the sign "x + y" is a sign
for the arithmetical function, addition, and substitution
of the numerals "2" and "3" for the variables yields
the sign "2 + 3" which refers to the number which is
the value of the function for the set of numbers (2,3),
namely, the number 5. A set of names or signs for
arguments which form an argument-set, is called an
"argument-name-set", or, more shortly "name-set".

5.B.3.c. When the domain of definition is restricted so
that some argument-places may be taken only by objects
of one kind, and some argument-places only by objects of
another kind, this may be indicated by a convention using
special kinds of variable-letters to indicate kinds of
arguments. (Cf. 5.A.13.)

Where the objects which are taken as values of a
function can all occur as members of the argument-sets
for which the function is defined, the function is
called an "operator," and its application may be
"re-iterated". (E.g. re-iterated application of addition
is symbolized thus: "x + (y + z)".) Where there is a
family of functions, such that the values of some may occur
as the arguments of others, and the values of the latter may be arguments for still others, etc., the functions may be applied *successively*, as is customarily indicated by such notations as "F(x, g(y, h(z, w)), u)" for the successive application of the functions F(x, y, z), g(x, y) and h(x, y). In this way more and more complex functions may be built up by successive application. The rules or techniques for finding values of such complex functions are derived from or constructed out of the rules or techniques of the component functions.

5.B.3.d. An example of a kind of function which is often an operator is what I shall call the "name-function" of a function. If "F(x, y, z)" is the sign for a function, then, as shown by the above remarks, there will always be another function, called the "name-function" for F(x, y, z), which takes as its argument-sets name-sets for the function F, and yields as its values the signs obtained by substituting names of arguments for variables in the sign for F. As the sign for the name-function, I write "/F/" or "/F(x, y, z)/", enclosing the name of the function between strokes. So, in this case, if "a", "b", "c" are names of arguments, then (a, b, c) is an argument-set for F(x, y, z), and ("a", "b", "c"), being the corresponding name-set, is an argument-set for /F(x, y, z)/, and its value, viz. /F("a", "b", "c")/, is the name "F(a, b, c)". [There is a whole hierarchy of name-functions, since to the function /F(x, y, z)/, there corresponds the name-function //F(x, y, z)//, taking for its values such signs as "/F("a", "b", "c")/", which, as just

1. See 5.B.3.b, above.
indicated, is a name for the name "F(a,b,c)."

Mathematicians and philosophers often confuse functions and their corresponding name-functions, and so also arguments and names of arguments, values and names of values. Sometimes this does not matter as the context makes clear what is meant. But it does matter when attempts are made to explain what propositions are in terms of the notion of a function. (Note: the name-function is not a sign for the name of a function. It is the sign for a rule which is applied by substituting names of arguments for variable-letters in the name of a function.)

5.8.4. Now let us return to the question: how do logical constants contribute towards the meanings of statements which employ them?

We may recall the fact, mentioned in 5.8.13, that the logical form of a statement, i.e. the way in which logical words and constructions occur in it, can be represented symbolically by sentence-matrix in which the non-logical words of the sentence expressing the statement are replaced by variable-letters. Thus, starting with the statement "Fido is a dog, and all dogs are four-legged", we obtain the matrix: "x is a P, and all P's are Q". We have here something strongly reminiscent of the notation for functions, and this tempted Frege, for example, to say that the original sentence must be the name of a value of a function. He wished to say that the thing named by the sentence (i.e. the value of the function for Fido, etc., as arguments), was a truth-value, the True or the False. This seemed odd, because what the sentence was a name of, i.e. its truth-value,
must depend on how things are in the world, whereas what we understand by the sentence (or by a name usually) does not depend on the facts, such as whether Fido really is a dog. Frege, of course, was not faced with this sort of difficulty, since he, unfortunately for logic, was interested mainly in mathematical propositions, whose truth-value does not depend on contingent facts.

5.B.5. One way out of the difficulty, would be to say that the sentence is the name of a proposition, which is the value of the function "x is a P, and all P's are Q" for the arguments (Fido, dog, dog, four-legged), but this would be of no use for our programme, since we are concerned to explain what propositions are, and so must not assume a knowledge of what they are.

I believe that we could regard a sentence as naming a class of possible states of affairs (possible states of the world). But I think there is a more illuminating way of looking at things, which makes it easier to explain how a statement may be true in virtue of its logical form, or in virtue of what it means. We may allow that a sentence corresponds to a truth value. But the meaning of the sentence, what is understood by it, is a method for discovering truth-values.

Meanwhile we may notice one thing. To every sentence-matrix, no matter what function or other entity it represents, there corresponds a name-function (see 5.B.3.d), which takes non-logical expressions as arguments, such as "Fido", "Willie", "The animal under the table", "dog", "horse", "four-legged", "hungry", etc., and yields sentences as values. Thus, the function /"x is a P, and all P's are Q"/ may take as a value the sentence "The
animal under the table is a horse and all horses are hungry". Sentences, therefore, may be regarded as values of name-functions.

5.B.6. Now in order to show how logical constants contribute towards the meaning of a sentence, I wish to introduce a new concept, the concept of a rogator, which is something like the concept of a function, but not quite. A function is a rule or principle which yields a value for an argument-set, the value being determined by the rule and the argument-set, whereas a rogator is something which does not fully determine the value but to which there corresponds a method or technique for discovering the value, which (i.e., the value) may depend on contingent facts having nothing to do with the rogator itself, or the principle on which it works.

A simple example of a rogator is the following, R(x), which takes bottles for its arguments and the sun, the moon or the earth for its values. In learning how to find out the value of the rogator for any particular argument, i.e. any particular bottle, one must learn to apply the following technique:

Examine the bottle to see whether it is empty or contains liquid, and, depending on whether it is empty, less than half full of some liquid, or half full or more, write down, respectively, "the sun", "the moon" or "the earth". What has been written down is then the name of the value of the rogator R(x) for the bottle in question (at the time of observation).

In this example, as in general, in order to know what the value of a rogator is for a given argument, one must know what the argument is (i.e. which object it is), one must know the general technique for determining the value, and one must know certain facts, or have performed
experiments. The argument and technique alone do not determine the value, for that depends also on what the facts are, and the value of a rogator for given arguments may change from time to time. (E.g. emptying a bottle may change its value for $R(x)$ from the earth to the sun.) A time-dependent rogator can always be turned into one which is not time-dependent by adding an argument-place for a time, or time indicator. It is important to notice that the technique for applying a rogator (for determining its values) may be learnt by example, and memorized, without the aid of an explicit description of the way in which it is applied. (See Appendix III.)

5.B.6. (note). Frege did not need to talk about rogators since he was concerned with mathematics, in which the values of functions are determined by general principles, independently of empirical facts. Of course, from a certain point of view, which takes account only of the way things actually are in the world, and not of what might have been the case, the notion of a rogator collapses into that of a function. But one cannot develop a complete theory of meaning without taking into account possibilities as well as actual states of affairs, since to understand a sentence is not merely to know whether it is true or false. (See 2.C.5 and 4.B.6.) This is why "extensional" systems of logic are of limited interest.

5.B.7. I wish now to describe a more interesting kind of rogator, illustrated by a game played with the aid of arithmetical symbols, and in particular the symbols for addition and multiplication, $(x + y)$ and $(x \cdot y)$.

The game is played as follows. A machine, or some
person (God) continually churns out little boxes, in each of which is a slip of paper with a numeral, the name of a positive or negative integer, such as "3", "-27", "3862". (0 is taken to be a positive integer.) On each box is written a letter or other sign, which is described as its "name", e.g. "A", "B", etc., there being no principle connecting the name on the box with the numeral inside it.

The players make their "moves" in turn, by selecting a name-function of some arithmetical function compounded by successive application (see 5.B.3.c.) of addition and multiplication [e.g. the function \( x \cdot (x \cdot x) \), or \( (5x + y^2z) \cdot (x + 3w) \), and so on]. This name-function is then applied to an argument-set consisting of an ordered set of names of boxes. Thus if a player selects the name-function \( /x + y \cdot z/ \), and the names, "A", "B" and "C", then he will make his move by reading out "A + B.C" or "A plus B times C".

Each move is awarded a tick or a cross, as follows. The boxes corresponding to the selected names are examined and the value of the arithmetical function worked out for the numbers referred to in those boxes as arguments. Thus, if the numerals in the boxes named are found to be "5", "-2" and "3", then the value of the function in the case illustrated will be \(-1\), that is, \(5 + (-2)3\). A tick is awarded if the value is positive, a cross if it is negative.

The next player then makes his move, in the same way, by selecting a function and set of names and "applying" the function to the names, being awarded a tick or a cross, depending on the results of examining the boxes so named and calculating the value of the function. (The player with most ticks is said to be "winning".)
5.3. This game provides us with a new kind of rogator, which takes boxes or names of boxes as arguments and ticks and crosses, or perhaps the words "tick" and "cross", as values. Though derived from arithmetical functions these rogators have different domains of definition, and different domains of values, from arithmetical functions, and they do not fully determine their values for given sets of arguments (for that depends on which numerals happen to be in which boxes).

Learning to play the game involves learning certain techniques, such as the technique of calculating the values of arithmetical functions for particular sets of numerical arguments. But this is not all. One must know how to decide whether a "move" is to be awarded a tick or a cross, and this involves knowing how, given the ordered set of names used in the move, and the function employed, to select the appropriate boxes, look at the numerals inside them, calculate the value of the function, and then say "tick" or "cross", depending on what comes out of the calculation.

We see therefore that a complicated technique must be mastered by anyone who wishes to play the game. It is a general or uniform technique, since new boxes are continually being produced, with new names on them and new numerals inside them, and one must know how to deal with whatever turns up, and not just how to work with the first twenty boxes which appear (e.g. by memorizing the numerals inside them, and their values for a certain set of functions). We can learn to apply such general techniques quite easily, for example by watching others and being given instruction in elementary arithmetic. We need not, however, either hear, nor be able to formulate, any explicit description of the techniques.
Thus, knowing which rogator is involved in a move, means knowing how to apply a general technique for awarding a tick or a cross, given a set of names. We may represent these rogators by means of symbols, such as "P + Q.R", or "5,F^2", etc., where "P" and "Q" etc., are variable-letters which indicate that argument-places are to be filled by names of boxes, and the whole symbol indicates which technique is to be applied for working out the value of the rogator.

To each rogator there corresponds a name-function (5.B.3.d.) very like that which corresponds to the arithmetical function from which it is derived except that one takes names of boxes (or names of names of boxes) as arguments, and the other takes names of numbers.

5.B.9. It should be clear now what I am getting at. Instead of regarding the symbols (sentence-matrices) which represent the logical forms of propositions as corresponding to functions (see 5.B.4), I shall regard them as corresponding to rogators, which will be described as "logical rogators". They may be represented by such symbols as "x is P", "All P Q's are R", "x is a Q and there are no R's which are S", etc. Corresponding to them are name-functions which take sentences as values. (See end of 5.B.5.)

5.B.10. We can regard rogators as taking either things or words which refer to them as arguments. For reasons of convenience of exposition I shall describe logical rogators as taking meaningful non-logical descriptive words and referring expressions as arguments, their
argument-places being represented by higher-case and
lower-case variable-letters, respectively. We could,
instead, talk about the things referred to as the
arguments. (A rogator, may, like a function, have a
restricted domain of definition. See 5.B.3.a and
5.B.3.c.) For the time being I shall take the words
"true" and "false" to be the values of logical rogators.
(But see 5.B.18, below.)

To each rogator there corresponds a general tech-
nique, which I shall describe as a "logical technique"
for determining its values, given an argument-set.
The technique involves looking at non-linguistic enti-
ties and then deciding to award the word "true" or the
word "false" for the "move" in language which is (or
would be) made by uttering the sentence obtained by
replacing variable-letters in the sign for a logical
rogator by suitable arguments for that rogator.

5.B.11. For example, for applying the logical rogator
"All P Q's are R", (derived from the logical form of
statements like "All red boxes are square"), we might
use the following technique. Given an argument-set of
three descriptive words, seek out the objects having the
properties referred to by the first two words, examine each
of them to see whether it has the property referred to
by the third descriptive word, writing down a tick if it
has, or otherwise a cross. When finished, look to see
if there is a cross amongst the things written down; if
not the value is "true" and otherwise "false".

In 5.A.3, we have already described the technique for
the use of the copula, in the rogator "x is P".

It is not essential that the techniques should be
described in these ways. There may be other techniques
with the same effect, and there may be various ways of describing the same technique. The important thing is that there are techniques which can be learnt, and which enable one, given a knowledge of the things (particulars or universals) referred to by non-logical words, to examine "the facts" or "the way things are in the world", and award truth-values to statements.

5.B.12. As before (see 5.B.8) generally applicable techniques correspond to each logical ro-gator. For in learning to use the logical form "x is a Q and there are no Q's which are R", it is not enough to learn to determine the truth-value when one of the words "Tom", "Dick" or "Harry" is taken as argument in the place of "x", and the other arguments are "man" and "happy". Nor is it enough to know how to find a truth-value for an argument-set as things actually are. One must know how to determine it for all suitable arguments in all possible circumstances, otherwise one does not fully understand. (Cf. 5.B.6.note.)

The techniques are, in fact, so general that it does not matter to which particular material objects the referring expressions correspond, nor to which properties (or "improper" properties) the descriptive words correspond. The techniques are topic-neutral. (See 5.A.3-4.) (We shall see later on, in Section 5.E, that this must be qualified.)

5.B.13. We are now almost in a position to say explicitly what must be learnt when one learns to use logical words and constructions. This requires a slight extension of the notion of a logical ro-gator and the corresponding name-function. (See 5.B.3.d.)
So far we have considered only name-functions which take sentences as values (see end of 5.B.5). But there are name-functions which take referring expressions or descriptive expressions as values. In addition, we must allow not only descriptive expressions and referring expressions as arguments, but also whole sentences. Thus, the name-function /"the R of x"/ takes relation words and referring expressions as arguments and yields referring expressions as values, such as "the father of Napoleon". The function /"P and Q"/ takes descriptive expressions both as arguments and as values, as in "red and round". The function /"∅ and ∅"/ takes sentences as arguments and yields sentences as values, such as "Fido is a dog and all dogs are four-footed".

Since the value of a name-function may be a sentence or a referring expression or a descriptive word, it may occur as the argument of another name-function. Thus, the function /"as P as x"/ may take as arguments "tall" and the expression mentioned above, and yield as a value the descriptive expression "as tall as the father of Napoleon".

Thus, by successive application of name-functions we can construct more and more complicated name-functions, just as in our arithmetical game more and more complex arithmetical functions could be constructed out of addition and multiplication, by successive application. (See 5.B.3.c.) For example, we get the name-function /"x is as Q as the R of z"/ by successive application of the functions /"x is P"/, /"as Q as z"/ and /"the R of y"/. Similarly, the logical form of a proposition may be regarded, often, as constructed by successive application of logical rogators to form a new, more complex rogator.
5.B.14. We have already noticed that a name-function which takes non-logical words as arguments and yields sentences expressing statements as its values may be thought of as corresponding to a logical rogator (a technique for determining truth-values). In addition, any of the name-functions described in 5.B.13 can be thought of as corresponding to a logical rogator, which takes as its arguments linguistic expressions of various sorts, and as its values either a particular object, or a property (proper or improper) or a truth-value, depending on whether the values of the name-function are referring expressions, descriptive expressions or sentences. For example, the logical rogator "the R of x" takes for its values particulars, such as the father of Napoleon, which are referred to by the values of the name-function /"the R of x"/. To each such logical rogator there corresponds a technique which must somehow be learnt for finding out which thing, property or truth-value (etc.) is the value of the rogator, given a set of arguments and their meanings. The technique, as before, must be generally applicable. (See 5.B.12.)

5.B.15. Now the role of logical words and constructions can be described explicitly: learning to use them involves learning to apply name-functions to non-logical words and expressions as arguments, obtaining referring expressions, descriptive expressions and sentences as values. (Formation-rules must be learnt.) Secondly, it involves learning how the meanings of the resulting combinations of words depend on the meanings of the expressions taken as arguments, or, more specifically, if the resulting expression is a referring one, one must know how to tell which is the object to which it refers; if
it is a descriptive expression, one must know to which properties it refers (or, more generally, how to recognize objects which it describes); if it is a full sentence, one must know which logical techniques to apply to the material objects, properties or other non-linguistic entities referred to by the expressions taken as arguments, in order to arrive at a truth-value. (I.e. one must learn which logical rogators correspond to which logical forms.)

The principles and techniques which must be picked up if one is to use logical constants may be very complicated and difficult to formulate explicitly.

5.3.16. I shall not try to describe in detail or classify the various rules for the use of specific logical constants (i.e. logical words and constructions) in all contexts. That is the task of the formal logician, and in any case it would be very complex since there is an enormous variety of cases and many intricacies would have to be taken account of, such as the fact that one and the same English word can correspond to functions and rogators taking various sorts of arguments and values. (E.g. "or" as a function of descriptive expressions in "red or round", and as a function of sentences in "That is red or that is round"; "is" of identity and "is" as a copula.) Moreover, the logical form of a proposition (or the corresponding logical rogator) may not be fully determined by its geometrical or syntactical form: other things, such as the context of utterance, or the type of entity referred to by one of the non-logical words may have to be taken into account. (Compare, for example, "I want that cake" and "I made that cake"; or "Fido is black", and "The dog you heard is Fido".) In consequence, difficulties arise if we try to represent logical form in
the usual way simply by removing non-logical words and replacing them by means of variable-letters.¹

5.B.17. I shall henceforth ignore the rules for the individual logical constants, discussing only the results of combining them with non-logical words and expressions to form whole sentences. So I shall discuss only the logical rogators or logical techniques which correspond to complete sentence-matrices (5.A.13, 5.B.9-10). All we need notice in connection with individual logical constants is that learning to use them involves learning very general principles for dealing with the non-logical words with which they may be combined, or which are taken as arguments for logical rogators. (See 5.B.12.) This generality, or topic-neutrality helps to account for the fact that we can cope with newly invented descriptive words (e.g. the name for the colour of a new synthetic dye) without formulating new rules for the use of logical constants in

¹ Some of these complexities are described briefly by Holloway, in "Language and Intelligence" (see pp.144 to 152). He seems to think that pointing out these irregularities demonstrates that signs in a calculus do not work like words in a language (p.144). But I have argued that there is a more fundamental reason, namely, that signs in a calculus simply cannot occur in true or false statements about anything. (See circa 5.A.6, above.) What the irregularities show, however, is that certain types of calculus do not provide adequate symbolic representations of forms of propositions, but this can surely be remedied at the cost of a loss of elegance by the choice of a calculus with suitably complex formation rules, rules of derivation, etc. (There would have to be different sorts of variables.) No such remedy can, however, turn a calculus into a language, for a language needs semantic rules as well as syntactical ones. (See section 2.D.)
sentences including them. It helps to explain how we are able to construct sentences to deal with totally new and unexpected situations. (However, see qualifications in 5.E.6,ff, below.) It explains how we can successively apply name-functions to form more and more complicated sentences, expressing more and more complicated propositions.

These general principles for dealing with new combinations of words, and for discovering truth-values in new conditions, all have to be memorized in learning to talk. (This does not mean that formulations of the principles have to be memorized.) The fact that they can be memorized is all that we need to explain the possibility of learning and teaching the use of logical words. (See quotation from Pap, in 5.A.2.) We certainly do not have to postulate the existence of any "if-feelings" or other peculiar subjective entities correlated with logical words.

(Though I do not plan to formulate principles for the use of individual logical words, I have already described explicitly the logical techniques which correspond to certain logical forms of complete propositions, such as "x is P" and "All P Q's are R", in 5.A.3 and 5.B.11.)

5.B.18. I have so far regarded the words "true" and "false" as the values of logical rogators corresponding to certain logical forms of statement, or sentence-matrices. This, however, does not explain how we can make statements to convey information, except by asserting something like: "The sentence 'The sun is shining' corresponds to 'true'". It is possible to modify my account of logical rogators by regarding them as taking
for their values, not truth-values, but sentences, namely the sentences obtained by applying their name-functions to argument-name-sets, in cases where the value would be true, or slightly different sentences including the word "not" where the value would be "false". (Compare: in our arithmetical game, instead of awarding ticks or crosses for "moves", the players might simply write down whatever is read out by a person in making a move in cases where it would be awarded a tick. E.g. they write down "A + B.C" when that move is awarded a tick. If it merits a cross (i.e. if the value of the arithmetical function turns out negative) they write down, instead, "(-1).(A + B.C)". The reader may fill in details of how this works in the game and what its point is.)

So in learning to talk we learn to utter sentences themselves in cases where our investigations and applications of logical techniques would yield the value "true". In cases where the value would be "false", we learn to prefix the sentence with the words "It is not the case that ...." or something similar, or, more commonly, to utter another sentence which would correspond to the value "true" (which, in many cases can be derived from the original one by suitable insertion or removal of the word "not"). Others, who have learnt to speak the same language, then, if they hear us and trust us, know what to expect when they look at the facts we have observed in applying the logical techniques.¹

¹ The rule might have been the other way round. We might have learnt to play the game in such a way that utterance of the original sentence took the place of utterance of the word "false", and utterance of the amended sentence took the place of the word "true", everything else being the same. In that case, sentences in that language would simply mean what their negations mean in ours. To utter a sentence in that language would still, of course, be to say what is the case, though we should take it to say what is not the case, since we should not understand.
5.3.19. We now see that whether a sentence $S$ expresses a true or a false proposition depends on three things:

(a) the logical form of $S$, which determines a logical rogator or general technique for determining truth-values, or for determining when $S$ may be uttered and when not;

(b) the meanings of the non-logical words and expressions taken as arguments (i.e. their semantic correlations with things and properties); and

(c) the way things are in the world, which is, in general, discovered by carrying out observations in the course of applying logical techniques.

(The fact that there is this third element shows that there is something right in correspondence theories of truth. Only by talking about rogators instead of functions can we bring this out.)

These three elements are all illustrated by our arithmetical game. For example, to three elements in the claim to know that some statement is true correspond three elements in the claim to know that a move in the game deserves a tick, thus:

(a) Knowing which boxes correspond to which "names" may be compared with knowing the meanings of non-logical words.

(b) Knowing how to look into appropriate boxes and apply the technique for deciding whether to award a tick or a cross corresponds with knowing the general principle or logical rogator determined by the logical form of a proposition.

(c) Looking at numerals actually written out in the boxes corresponding to names used in a move in the game, is like observing the facts which determine whether a sentence expresses a true or a false proposition.

The analogy should not be taken too seriously. I have tried to find a comparison which is close enough to serve my purposes without being too close to be illuminating: very likely an impossible requirement!
5.B.20. I have tried to describe the role of logical constants in giving sentences their meanings by saying that the way they occur in sentences determines which logical rogators correspond to the propositions expressed by those sentences. Thus they help to determine the kind of technique which must be employed in discovering whether a proposition happens to be true or not. So they help to determine which sorts of facts can count as verifying the proposition, or in which possible states of affairs the proposition would be true. Logical constants do not indicate which entities or kinds of entities a proposition is about (they are topic-neutral—see 5.A), but they do indicate how things must be with those entities if the proposition is true. (I have ignored the role of logical constants in questions, commands, and so on, but I think this could easily be taken account of.)

It might be thought that this is an unnecessarily long-winded and round-about way of describing how propositions work. Certainly there are alternatives. For example, we could regard sets of non-logical words as determining functions and the logical words as the arguments to which they are applied. (In general, there is a symmetry between argument-sets and functions.) Or we might try to avoid mentioning rogators and manage with functions alone, as Frege and Wittgenstein and Russell did. Notice, however, that my method eliminates many obscurities in their accounts. Thus, there is no need to discuss Frege's "unsaturated" entities of which nothing can be predicated. Nor need we talk about "unsayable" facts which merely "show" themselves, though, as will appear in chapter seven, in connection with knowledge of necessity,
this is only a temporary advantage. The price which we have had to pay for explicitness is, of course, circularity. But I hope the circle is so wide that this does not matter.

5.B.21. This completes the account of the way in which logical constants help to determine the truth-conditions of propositions which they are used to express. (All this shows, incidentally, that the employment of verbs is not essential for the activity of statement-making. Verbs have a special function which need not be described here.) Now it remains to show how this sheds light on the existence of logical properties of propositions or logical relations between them. We must try to understand, for example, how it is possible for propositions to be true, or inferences to be valid, in virtue of their logical form. Once again, the analogy of the arithmetical game will be useful.

5.C. Logical truth

5.C.1. Logical properties of propositions and logical relations between propositions can be shown to be due to relations between the logical rogators involved in their construction. I shall first of all illustrate, using the example of the arithmetical game (see 5.B.7.), the general way in which rogators may be related and then turn back to logical rogators and propositions.

5.C.2. It will be recalled that "moves" are made in the game by applying rogators, derived from arithmetical functions, to names of boxes as arguments, ticks or crosses being awarded according as the arithmetical functions have
positive or negative values for the numerals in the boxes named. The first thing to notice is that relations of "entailment" may hold between moves in the game.

Consider moves made with the two functions $f(x,y)$ and $g(x,y)$, where the former is $x^2 + (-3).x.y$ and the latter is $x^2 + (-3).x.y + 27$. The rogators derived from these functions are $f(P,Q)$ and $g(P,Q)$, taking names of boxes as arguments. To each rogator there corresponds a general technique for working out its value for any argument-set, the value being "tick" or "cross" depending on which numerals are found in the boxes referred to. (See 5.B.3). Now, since the value of the function $g(x,y)$ is always greater than the value of $f(x,y)$ by 27, for the same arguments, owing to a relation between the techniques for calculating their values, it follows that the former is positive whenever the latter is. Hence, if any move made with $f(x,y)$ and names "A" and "B" is awarded a tick, then, in those circumstances, a move made with $g(x,y)$ and the same names would also merit a tick.

Owing, therefore, to a relation between the techniques for finding their values, the two rogators $f(P,Q)$ and $g(P,Q)$ are themselves related so that the value of the latter for a pair of names of boxes as arguments must be "tick" whenever the value of the former is, for the same arguments (in the same circumstances). Of course, the value of $f(P,Q)$ might be "cross" when the value of $g(P,Q)$ was "tick", but the converse could not happen. We can tell, merely by examining the techniques corresponding to the two rogators, without looking to see what is in the boxes, that if the value of $f$ is "tick", then the value of $g$ is also "tick". We may say of a
pair of moves made with these rogators, such as
"A² + (-3).A.B" and "A² + (-3).A.B + 27", that the
former "entails" the latter.

5.0.3. Now we must notice what happens when complex
rogators are constructed out of rogators between which
relations hold. Since the value of the arithmetical
function g is greater than the value of f for the same
arguments no matter what they are, it follows that the
arithmetical function g(x,y) + (-1).f(x,y) is positive
for all values of the arguments. Call this function
h(x,y). To it, as usual, there corresponds a rogator
h(P,Q), taking names of boxes as arguments, but with the
peculiarity that the value of the rogator for all argu-
ments is "tick" in all circumstances. The general
technique for discovering the value ensures that all
moves in the game made with h and any pair of names of
boxes must be awarded a tick, no matter what numerals
are in the boxes, and this may be discovered simply by
examining the technique for working out values of this
rogator.

Of course, this is not the only way in which a one-
valued rogator can be constructed. Other examples are
the rogators derived from the following arithmetical
functions:

x.x, x.y.x.y, and x.x + y.x + y.y

each of which has positive values for all arguments.

5.0.4. All this shows that although in general the value
of a rogator for a given set of arguments has to be
discovered by looking at the facts and applying the
general technique for determining its value, there are,
nevertheless, some cases where a complex rogator, com-
pounded out of simpler ones (see 5.8.3.c, 5.8.13) by
successive applications, is a "freak" in that one can
discover its value merely by examining the technique
for working out its value, or by examining the tech-
niques of the simpler rogators out of which it is
compounded. Similarly, by examining the techniques for
working out the values of a pair of rogators, one may
discover that they stand in some "internal relation"
so that knowing the value of one of them may enable one
to work out the value of the other without consulting
the facts or actually applying its general technique.

5.0.5. In some cases we can look at the way in which
the rogator determines its value independently of the
facts slightly differently. For example, in the
arithmetical game, the use of the function \( u \cdot v + w \cdot x + y \cdot z \)
in a move may result in a tick or a cross being awarded,
depending on which names are used and which numerals are
in the boxes corresponding to those names. But if we
take the argument-set consisting of the names "A", "A",
"B", "A", "B", then we obtain the move "A.A + B.A
+ B.B", which gets a tick no matter what is in the boxes,
since \( x^2 + y \cdot x + y^2 \) is positive for all values of \( x \) and
\( y \). This shows that in some cases we can look at the
value of a rogator as determined not only by properties
of the technique for working out its values, but also by the
"structure" of the argument-set. In these cases, by
examining the technique for determining values of the
rogator and the structure of the argument-set, we can
find a value which it must take for all arguments with
that structure, no matter what the facts are, although in
general the result of applying the techniques corres-
ponding to the rogators out of which the "freak" is con-
structed does depend on the facts, that is, on how things
happen to be in the world. (See 5.B.6.)

5.C.6. All this applies to logical rogators as well as the ones which we have been discussing: logical rogators may also generate "freaks". We saw (in section 5.B) that every sentence can be thought of as the result of applying a logical name–function to a set of non–logical words, the logical rogator corresponding to that name function being what determines the conditions in which the sentence so obtained expresses a true proposition. As in the cases discussed above, it may be possible, by examining the general techniques for determining the value of a logical rogator, to discover its values for all arguments, or for argument–sets with certain structures, without consulting the facts at all. So we can determine the truth–values of propositions constructed with the aid of such rogators merely by examining the logical techniques for discovering whether those propositions are true or false, i.e. without applying the techniques.

For example, the sentence "All red horses are red" is a value of the logical name–function/" All P Q's are R"/ for the argument set ("red", "horse", "red"). To it there corresponds a logical rogator and a technique for determining truth–values. By examining that technique, and the structure of the argument–set, we can discover the truth–value in question without actually applying the technique (which would involve examining all red horses to see whether they are red). We can discover that the proposition expressed by the sentence is true independently of the facts, and independently of the actual meanings of the words in the argument–set (as long as the structure is ("A", "B", "A")). We say that it is
a "formal" truth, true in virtue of its logical form.

5.C.7. It is extremely important to notice, in all these cases, that where the value of a rogator is determined independently of the facts, this has to be discovered by examining the technique, not by applying it. But since one may have mastered a technique without ever examining it (see appendix on "Implicit Knowledge"), one may fail to notice that a rogator is a "freak" whose values are determined independently of the way things are, and go on as usual to find out its value by applying the technique. (It should be recalled that the techniques are generally applicable: they work for all argument-sets, even those whose structure restricts the possible outcome of applying the techniques. See 5.B.3, 5.B.12, 5.B.17.)

So, in the case of the game, the players may fail to notice that a move such as "A.A + B.A + B.B" would merit a tick no matter what numbers were found in the boxes referred to, and go on in the usual way to look into the boxes, calculate the values of the arithmetical functions, and base their decision whether to award a tick or a cross on the result of applying this technique. Similarly, one may fail to notice that some proposition is true in virtue of its logical form, and apply the usual logical techniques for determining its truth-value by observation. This is possible because the techniques are generally applicable. Thus I can discover that "All the red horses in this room are red" expresses a truth by examining the red horses in the room (cf. 5.B.11), but there is no need to, since I can see what the outcome would be merely by thinking about the method which I should have to apply.
(See 5.4.9 for another example.) The importance of this will emerge in section 6.5 on "Knowledge of analytic truth".

5.0.3. All this can be extended to explain the existence of logical relations between propositions, arising from relations between their structures. For example, we may learn that one proposition entails another in the same sort of way as we found in 5.0.2. that one move in the game could "entail" another. We may find, by examining the logical techniques for discovering truth-values of the two propositions, and the structures of their argument-sets, that no matter how things are in the world, if the outcome of applying one of these techniques is "true", then so will the other be. E.g. If "All black horses are hungry" expresses a true proposition, then so does "All big black horses are hungry". In such a case, we may speak of "formal entailment". The inference from one proposition to the other will be "formally valid", or valid in virtue of its logical form. (The logical form of an inference can be represented by substituting variables for non-logical words, in much the same way as the logical form of a proposition.

(Cf. 5.4.13.) Similarly, propositions may be formally contradictory, or formally incompatible. As with formal truth, such logical relations may pass unnoticed by persons who apply logical techniques without examining them. (We cannot give a general definition of "entails" until after chapter seven.)

5.0.9. I wish to stress the (by now obvious) point that these logical properties and relations of propositions are not due merely to geometrical relations between symbols,
but primarily to properties of and relations between techniques which have to be learnt for doing things with these symbols. Admittedly, in most languages the rules for the use of symbols are probably so chosen that to certain geometrical relations there correspond relations between techniques (as implied by my remarks in 5.B.14–15 about the connections between name-functions and logical rogators). This is indispensable if there are to be general principles for constructing more and more complicated types of propositions out of a small set of symbols without continually introducing new ad hoc grammatical and logical rules of construction: to this extent logic may be connected with syntax, though it is never reducible to it. However, as remarked in 5.B.17(note), not all rules of formation of sentences are quite like this, so the connections between logical techniques and geometrical forms are not absolutely indispensable and, in any case, it is not enough to notice the connection between geometrical relations and logical relations. Indeed, noticing this may blind philosophers to the intermediary in virtue of which they are connected, with unfortunate results, as I shall try to show in Appendix II.

Part of the explanation of the tendency of philosophers of logic to ignore these logical techniques, is the fact that we can learn to use symbols and apply the corresponding techniques, and sometimes even draw consequences from the interconnections between these techniques, without fully realizing what we are doing. We need not even be aware of the existence of the techniques. This is an illustration of a general point that one may have knowledge which one cannot formulate, or one may know that something is so without being quite
aware of the reasons why they are so or how it is that one knows this. One may claim, with perfect justification, to know that "If anything is red then it is red" expresses a truth, and yet be completely inarticulate when there is any question of justifying the claim. (This sort of thing is discussed in Appendix III, and in 6.E.6.) I have been trying to make the missing justification explicit, or at least to describe it in general terms.

5.C.10. I have not, however, fully explained how we can draw conclusions about the outcome of applying certain techniques merely by examining those techniques, without actually applying them. I have not explained what goes on when one has the kind of logical insight which is involved in perceiving that two logical techniques are related in certain ways, or that a logical technique has certain properties, apart from relating it to the general way in which one may discover properties of or connections between regators.

Some would probably try to reduce logical insight to a matter of seeing that a certain sequence of formulae or symbols have certain syntactical properties, but this would leave unexplained the kind of insight one has when one sees that this is the case, which is a sort of mathematical insight into the connections between geometrical forms. In any case, we have already argued against attempts to reduce logic to syntax. (See section 5.A and Appendix II.) (For some reason it was only after the discovery of Gödel's famous incompleteness theorem that some logicians began vaguely to appreciate this point. Gödel expressed it as follows, in his contribution to
"The Philosophy of Bertrand Russell", p.127-8: "It has turned out that ... the solution of certain arithmetical problems requires the use of assumptions essentially transcending arithmetic i.e. the domain of the kind of elementary indisputable evidence that may be most fittingly compared with sense perception." I do not wish to say that this transcends arithmetic; I should rather say that it turns out that arithmetical knowledge requires more than was once thought by some logicians to be required. It transcends their conception of arithmetic.)

5.C.10.a. It cannot be argued that when we have this sort of insight or draw the sorts of conclusions under discussion, by examining the structures of argument-sets and the techniques for determining the values of logical rogators, what goes on is that we consider statements describing these techniques and structures and then apply some formally valid procedure for inferring, via formal entailments, that certain statements have certain logical properties or stand in logical relations. That would clearly be circular, since it is the nature of formal validity that we are trying to explain by talking about these properties of logical techniques. We cannot without circularity explain this by assuming that their having these properties is merely a formal consequence of other facts. (Cf. 7.D.9,ff.)

5.C.10.b. It seems to me that what goes on when we have this sort of insight, and in general when we discover facts about the application of techniques by examining them instead of applying them, is essentially the same sort of thing as goes on when we discover necessary
connections between, for example, geometrical structures or properties, by examining those structures or properties and perhaps constructing informal proofs. The difference lies in the degree of generality. (Logic is topic-neutral: see section 5.A.) This sort of thing will be discussed in more detail in the section on "Informal proofs", in chapter seven.

5.D. Some generalizations

5.D.1. Let us leave aside questions about what goes on when we examine logical and other techniques instead of applying them, and consider how some of the remarks of the previous section may be generalized, so as to prepare the way for the discussion of propositions which are analytic, or true by definition.

We have seen that although in general the value of a rogorator for any argument-set depends on how things are in the world, nevertheless there are some "freak" cases where the value can be discovered independently of observing facts and applying the technique for that rogorator. In some cases we found that what determined the value independently of facts was an interaction between the general technique for discovering values, and the "structure" of the argument-set. (See 5.C.5.) Let us look a little more closely at this sort of case.

It is clear that when the logical form "All P things are Q" is applied to the argument-set ("red","red"), the basic reason why the proposition expressed by the sentence so obtained is true independently of what is the case in the world, is not the fact that the two words in the argument-set are identical, but that they have the same meaning, that they refer to the same property.
This is seen from the fact that if we define a new word, "rot", say, to refer to the same property (proper or "improper" property\(^1\)) as "red", then applying the logical form to the argument-set ("red","rot") will yield the statement "All red things are rot", which is true independently of the facts for much the same reason as the original one. So our description of the original case was not sufficiently general. The fixed truth-value of a statement like "All red things are red" is not essentially due to the fact that it is obtained from an argument-set with the structure ("P","P"), but to the fact that it is obtained from a set of two arguments standing in a certain relation (in our example the relation is synonymy: the words refer to the same property).

5.D.2. We see therefore, that when the value of a rogator for certain arguments is determined independently of the facts, this may be due to (a) the general technique for discovering its values (e.g. the way the rogator is constructed out of other rogators), (b) the structure of the argument-set, and (c) relations between the arguments. (The second is really a special case of the third.) In this sort of case, the value of the rogator is always the same for a set of arguments standing in the appropriate relations, no matter what the arguments are, and no matter how things are in the world. (As before (see 5.C.7), a person may fail to notice this interaction between a rogator and an argument-set, and work out the value in the usual way by applying the technique as if its outcome could depend on the facts.)

5.D.3. Once again we can find an illustration in our arithmetical game. (5.B.7.) If it is known that no

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numeral occurs in more than one box in the game and that every numeral will eventually occur in at least one of the boxes, then we might, in the course of playing the game, introduce new names for boxes, as follows. If "B" is known to be the name of a box, then we say: "Let 'A' be the name of whichever box contains the numeral obtained by adding three to the number referred to in 'B'". We shall not, of course, know which box is the one referred to by the name "A", but we do know that whichever it turns out to be, the numeral in it will be in the stated relation to the numeral in B, whatever that may be. By considering this fact, and by examining the technique for deciding whether moves are to be awarded ticks or crosses, we can tell without looking into boxes that the move "A + C.C + (-1).B + C" must be awarded a tick since the value of the arithmetical function \( x + y.y + (-1).z + w \) must be positive for all argument-sets in which the second and fourth arguments are the same, and the first argument is greater than the third.

5.D.4. This example illustrates the same sort of thing as may happen when we apply a logical form to descriptive words whose meanings stand in some complicated relation owing to the fact that they have been logically synthesized in the manner described in section 3.B. Relations between meanings of words in virtue of which the value of a rogator is independent of the facts need not be as simple as in the example of 5.D.1, where the relation was synonymy. For example, if the word "U" refers to the property P, and the word "V" refers to the combination of properties Q and R, while the word "W" describes objects if and only if they have the property P or do not have the property Q, then the result of applying the name-function /"No F are G
and not $M'$ to the argument-set ("$W$", "$V$", "$U$") is the sentence "No $W$ are $V$ and not $U$" which can be seen to express a proposition whose truth is independent of how things are in the world, owing to (a) facts about the logical rogator corresponding to its logical form, and (b) relations between the meanings of the words.

Since the proposition is not true merely in virtue of its logical form (not all propositions with that logical form are truths), but also in virtue of relations between the meanings of some of the non-logical words, the proposition is not a "formal" truth. (See 5.C.6.) (We could alter customary philosophical usage and extend the notion of the "logical form" of a proposition to include such facts about the logical relations between meanings of non-logical words, and this would be a good thing insofar as it drew attention away from syntactical properties of sentences, but I shall not do so.)

5.D.5. We can now see that the fact that if words are given meanings standing in certain relations then this may have the consequence that some sentences in which they occur express propositions whose truth-values are independent of the way things are in the world, is just a special case of a more general fact about rogators, namely that relations between arguments in an argument-set together with properties of the general technique for discovering values of the rogator may in some cases suffice to determine the value for that argument-set, though in general it does not, since facts are relevant too. This does not mean that the general technique cannot be applied in order to discover the value in such freak cases, but that it need not be. (Cf. 5.C.7.) (We shall see later on that even an analytic proposition
can be verified by empirical observations, though it need not be.)

5.D.6. It should be noted that when there are relations between the meanings of descriptive words, although this does not enable us to discover the truth-values of all propositions which they may be used to express, there will certainly be a great many whose truth-values are determined. Thus, from the fact that the word "red" refers to the same property as "rot" we can infer not only that "All red things are rot" expresses a true proposition (See 5.D.1.), but also that each of the following does: "Nothing is both rot and not red", "If anything is not red then it is not rot", "All rot and round things are red or the moon is made of green cheese", etc. In addition, the following are false independently of the facts: "Some red things are not rot", "All round things are red and not rot and there is at least one round thing", etc. (To any one relation between the meanings of descriptive words, there corresponds a whole family of analytic propositions. See 6.F.5.)

In all these cases we can discover the truth-value in essentially the same way as before: namely by studying the general logical techniques which would normally be applied in finding out the truth-values of statements made with these logical words and constructions.

In addition, we could discover, by examining the logical techniques and relations between meanings, that certain relations such as entailment and incompatibility hold between some propositions. (This is just an extension of the remarks in 5.C.8.)
5.E. Conclusions and qualifications

5.E.1. The time has come to summarize what has been done in this chapter, and show how it fits into the general programme of this thesis.

The aim of Part Two, which this concludes, was to describe the general connection between meaning and truth, in order to prepare the way for a description of the connection between meaning and necessary truth, in Part Three. (We have been mainly concerned with statements containing only logical words and descriptive words referring to universals, but many of the remarks of 5.B apply also to statements in which particulars are mentioned.)

The general connection can be summed up thus: learning the meanings of words or sentences containing them involves learning to recognize or pick out states of affairs in which to utter such sentences is to make true statements. I have tried to isolate out two aspects of this learning process. First we have to learn semantic correlations between non-logical words and non-linguistic entities, and secondly we must learn the use of logical words and constructions. 1 (1) Semantic correlations between descriptive words and universals (observable properties and relations) were described in chapters three and four. Learning these involves learning to recognize the particular objects which may be correctly described by such words. (2) Learning to use logical words and constructions involves learning to tell which logical rogator (which generally applicable logical technique for determining truth-values) corresponds to the way in which logical words and constructions occur in a sentence. It is in virtue of this correspondence
that the occurrence of such logical constants helps to
determine the conditions in which the proposition
expressed by the sentence is or would be true (or false).

All this showed how the truth-value of a proposition
expressed by a sentence containing logical words and
descriptive words depended on (a) the meanings of the
descriptive words, (b) the logical techniques corres-
ponding to the logical form and (c) the facts, i.e. how
things are in the world. (5.B.6, 5.B.19.) We saw
that this was just one instance of the general fact that
the value of a rogator depends on (a) the objects taken
as arguments (b) the general technique (or rule, or
principle) for discovering values, and (c) the way things
are in the world.

This completed the account of the general connection
between meaning and truth.

5.B.2. Further investigation showed that the existence
of formal truths and formally valid inferences could be
explained in terms of properties of and relations between
rogators. (See 5.C.6, 5.C.8.) This eliminated the
need for making obscure, misleading or false remarks about
the connection between logic and syntax. (See section
5.A.) As remarked in the previous paragraph, the value
of a rogator depends, in general, on three things, but we
found some "freak" cases in which the third element dropped
out. In these cases, although the value could be dis-
covered by applying the general technique and investigating
facts or conducting experiments, nevertheless this was
not necessary, since the value could be determined a
priori.

We distinguished three types of freak case. (1) In
the most general case, the value depended on both of the
first two factors (namely (a) and (b) above), and could be discovered by examining the argument-set and the technique for discovering values of the rogator. The value was determined by relations between the arguments together with properties of the general technique, independently of the facts. It did not matter which particular objects were taken as arguments: the value was always the same, provided that they were related in a certain way. (Section 5.D.)

(2) A simpler type of freak rogator was one whose value was the same for all argument-sets with a certain structure. Here the value could be discovered by examining the structure of the argument-set and the general technique for finding values of the rogator, independently of how things were in the world, or which particular objects were taken as arguments.

(3) In the simplest sort of case, the value was completely independent of which objects were taken as arguments, and was fully determined by the general technique. Here both the first and the third factors dropped out, leaving only the second (b).

The second and third type of freak rogator sufficed to explain the existence of propositions true in virtue of their logical form, since such propositions corresponded to logical rogators constructed in such a way that their values for some or all arguments might be determined independently of the facts. The first type will be used to explain the more general fact that there are propositions which are analytic, that is true in virtue of the meanings of words, or true by definition.

A slight modification, taking account of relations between rogators, due to relations between their general techniques and their argument-sets, serves to account for logical relations between propositions, such as entailment,
incompatibility or logical equivalence.

5.E.3. We see from all this that it is possible to give an account of logically true propositions which arises naturally out of a description of the general connection between meaning and truth, covering contingent propositions too. There is no need at all to explain away logical truths as not being truths at all, or logically true propositions as not being propositions at all, but rules or conventions or expressions of acceptance of conventions, etc. They are propositions, and their truth-values may be discovered empirically by applying the general logical techniques for discovering the truth-values of contingent propositions expressed by sentences including the same sorts of words and constructions. Their peculiarity is only that their truth-values may also be discovered in the other way which I have described. (Cf. 6.E.1, ff.)

5.E.4. It is probably obvious that what I have said is closely related to Wittgenstein's explanation of logical truth in "Tractatus Logico Philosophicus". (He too was not content to classify and describe logical properties of propositions and relations between them, but tried to explain them.) His account, however, seems to me to have involved some unnecessary obscurity, and was certainly not sufficiently general. I have tried to give a more general account of logical form (of propositions containing logical words, descriptive words correlated with observable properties, and words referring to particular material objects). The logical form of a proposition corresponds to the way in which the truth-conditions of the proposition are related to the entities,
such as material objects or properties, mentioned or referred to in the proposition. Knowing the logical form involves knowing in general how to tell whether propositions with that logical form are true or false, no matter what entities are referred to, and no matter how things are in the world. (But see qualifications below.)

This is what people are talking about when they refer to the "real" logical form of a proposition, contrasting it with the "apparent" logical form suggested by the verbal form of a sentence. Of course, every intelligible sentence must fully determine the real logical form (otherwise it could not be understood correctly: we should not be able to recognize its truth-conditions). It is only when instead of applying the logical techniques we reflect on the logical form that the form of the sentence can suggest anything misleading to us. This, however, is a failing on our part, due to our not thinking clearly about what, in a way, we know quite well (see appendix on "Implicit Knowledge"), and does not mean that there is anything inaccurate or imprecise about the sentence. (E.g. though we know quite well the difference between the copula and "is" of identity we may get muddled when talking about it.)

5.5.5. It should be emphasized that I am not talking about a "perfect" language, or any one particular language. I have been trying to bring out general facts about any language which can be used for making true or false statements about material things and their properties, about the way things are in the publicly observable world. Neither do I restrict my remarks to sentences in some
special notation or "canonical form": what I say is intended to apply to all sorts of statements using all sorts of logical constructions, provided that it is possible to think of the statements as built up out of parts which have a general use in statements. (A language in which there was just one sound, which had to be learnt separately, corresponding to each statement, would be very different from ours. There would be no way of talking about possibilities, or of teaching the meanings of false statements - and my remarks would probably not apply in that case.)

5.B.6. Despite its generality, my account of logical form has involved a number of over-simplifications, which must now be eliminated. The first oversimplification is concerned with presuppositions. I have continually stressed the fact that to every roogator discussed so far there corresponds a generally applicable technique for discovering its value for all permissible argument-sets, which works in all possible circumstances (Cf. 5.B.8 and 5.B.12). This must now be qualified, for there may be some techniques which are applicable only when certain conditions are satisfied. For example, we described an arithmetical game (in 5.B.7) which involved techniques for working out the values of roogators taking names of boxes as arguments. Those techniques involved looking into boxes and working out the values of arithmetical functions taking the numerals in the boxes as arguments. If, however, a box were found to contain no numeral (e.g. there might be an apple in it instead) or two different numerals, then the technique could not be applied, and there was nothing in the rules of the game to say how to deal with this case. The rules (as I described them) did not say whether a
move using the name of a box with an apple in it should be awarded a tick or a cross or what. They merely took it for granted that the question would not arise.

5.B.6.a. Similarly, the applicability of logical rogators presupposes the satisfaction of certain conditions. Thus, the logical technique corresponding to the logical form "All \( P \) Q's are \( R \)" as described in 5.B.11 presupposes that there are no objects which are borderline cases for the descriptive words taken as arguments. (Recall the various sorts of indefiniteness of meaning of descriptive words described in chapter four.) If some boxes turn up which are neither definitely scarlet, nor definitely not scarlet, then that technique provides us with no way of assigning a truth-value to the proposition expressed by "All scarlet boxes are red". It should be noticed that the technique does not even provide us with a value for "All scarlet boxes are scarlet" in this case. Of course, an examination of the logical technique and the structure of the argument-set would, as described above, lead us to say that the truth-value must come out to be "true". But this presupposes that the technique yields a value at all, that its applicability-conditions are satisfied. (Compare the case where a player makes the move "A.A + 3", and the box corresponding to "A" has only an apple in it: he gets no tick although \( x^2 + 3 \) is always positive.)

So, had we been more precise and explicit, we should continually have had to make qualifications of the form: "... provided that the applicability-conditions of the technique are satisfied". These were omitted in the interests of clarity and simplicity (see 5.B.12.).
5.E.6.b. One kind of presupposition which has drawn some attention is that if a non-logical word or expression occupies an argument-place intended for an expression which refers to some one entity of a certain kind, then there is exactly one entity referred to and it is of the correct sort. For example, the use of the logical form "x is P" presupposes that the expression taking the place of "x" refers to a particular object, and if there is not exactly one to which it refers, then the technique (described in 5.A.3) for determining a truth-value lacks application. Hence that technique does not provide us with a truth-value. (Section 2.D was concerned to show that the same applies to descriptive expressions substituted for "P").

5.E.6.c. To sum up: just as functions and rogators may have restricted domains of definition, that is the classes of argument-sets for which they yield values may have certain limitations, so may there be restrictions on the class of states of affairs in which the techniques for determining their values can be applied. The domain of applicability-conditions may be restricted. In many cases, whether the technique yields a value or not, i.e. whether one of its applicability-conditions obtains or not, will depend on how things happen to be in the world (e.g. on whether there happen to be any borderline cases of instances of the colour scarlet, or whether there happens to be no king of France). But there are probably some cases where, from the way in which a rogator is constructed, and from facts about the things taken as arguments, one can discover without trying to apply the technique that it cannot yield a value for those arguments. That is to say, it may be impossible for the applicability-
conditions of some complex rogator to be satisfied when
certain things are taken as arguments. A detailed
investigation of such things as limitations on domains
of definition and restrictions on applicability-condi-
tions of the techniques corresponding to logical rogators
would, I think, shed a great deal of light on the subject
of so-called "category mistakes", and, in particular,
show how they differ from straightforward contradictions.

5.E.7. We see therefore that there are various ways
in which even a logically well-formed sentence may
fail to express a true proposition. The logical tech-
nique corresponding to it may yield the value "false", or
it may yield no value, for any of several different
reasons. This makes it look as if in some cases it is
correct to say that the proposition expressed by the
sentence is neither true nor false, or that no proposition
is expressed at all. But things are not quite as simple
as this, for, just as the semantic correlations between
descriptive words and properties may be indeterminate,
giving rise to difficult borderline cases, so may the
rules governing the use of logical forms and the prin-
ciples for deciding on truth-values be indeterminate,
giving rise to difficult borderline cases.

5.E.7.a. For example, superimposed on the principle
for determining the value of a logical rogator for given
argument-sets, may be a general principle of the form:
"When the technique does not (definitely) yield the value
'true', then the value is 'false'." If this principle is
added to the original one (for example to the rules for
"is", "or" and "all" described in 5.A.3 and 5.A.11), then,
when the applicability-conditions of the original technique
are not satisfied, this new general rule ensures that the truth-value is false. But in that case trouble arises from the fact that normally when the truth-value of a proposition is "false", there is a proposition derived from it by the insertion of the word "not" into the sentence expressing it, namely, its negation, or one of its contraries, which has the truth-value "true". And usually the applicability-conditions for the technique corresponding to the new proposition are the same as for the old one. Hence there is a conflict in cases where applicability-conditions are not satisfied, between what these rules lead to and what we should normally expect, and there may be no rules which are definitely part of the language to specify what is to be said in such cases.

This can be summed up by saying that there may be more than sufficient rules for the use of logical forms, which work in most cases but may come into conflict in others, and then there may be no definite answer to the question: "Is this proposition true or false?". Various rules for the use of logical constants are superimposed in an indeterminate way (Cf. 7.D.11, note.). Failing to see that this is a case of indeterminateness of linguistic rules, philosophers may argue in vain that one or other answer or some third one is correct. (See 6.D.4.) (Such controversies are not, of course, completely useless, since they help us to see various ways in which the principles governing the use of logical operators can be made more definite. See Appendix IV.)

5.E.7.b. Other kinds of indeterminateness in the principles governing the use of logical forms arise out of the fact that as a language develops, different ways may be found for saying the same thing, and this may involve extending
or changing the functions of logical and other words. For example, instead of saying "All P things are Q", we may learn to say "The class of things which are P is included in the class of things which are Q", or "The property P-ness is always accompanied by the property Q-ness" or even "P-ness is possessed only by things which have Q-ness". In this sort of way abstract substantives referring to universals are allowed to enter into sentences as if they had the same grammatical roles as words referring to particulars. We learn to say things like "Red is a colour", "Idleness is annoying", and these sentences strongly resemble "Fido is a dog" and "My table is brown", whose logical form is represented by "x is P".

It looks therefore as if the domain of definition of the logical rogator corresponding to this form has been extended so as to include new argument-sets. We cannot, however, simply say "Let there be an extension", for the technique for determining truth-values must be extended too. This extension enables us to apply the form "x is P" to argument-sets like ("my table", "brown") and ("brown", "a colour") or ("brown", "attractive"). This may lead us to think that we have extended the technique to cope with argument-sets like ("brown", "brown") even when we have not done so. Again, there may be conflicts between rules, with nothing definite in the language to settle them. We can, of course, extend the technique to cope with this kind of argument-set if we wish to do so, for we can give any form of words a use if we wish, but we are tempted to think that we have extended the technique and the domain of definition before we have done so in fact, or that we are compelled to extend it in a certain way, and this may lead us into
difficulties such as Russell's paradox, and others. (See what happens when people forget that division by zero is not defined in arithmetic. We could extend the notion of division to include division by zero, but if we do not wish our rules to lead into conflict some other changes may have to be made. (Rules lead to conflict when, for example, they enable a rogator to have more than one value - e.g. both "true" and "false" - for the same argument-set.)

5.E.7.c. A similar sort of indeterminateness arises when we use logical constants in talking about quite new kinds of things, such as infinite sets, thinking that their use is fully determined in these contexts by the rules for their use in other contexts. People may even disagree about the way in which the use is determined in the new contexts, failing to notice that this is a case of indeterminateness, where some new convention must be adopted if the matter is to be settled. (Cf. Section 4.0 and 7.D.10,ff.) So philosophers of mathematics may disagree as to how the logical constants are to be used in connection with statements about infinite sets, without realizing that there are alternative ways of using them, either of which may be freely chosen. (Or it may not definitely be the case or not the case that either can be freely chosen: the rules of the language in question may not definitely leave the matter quite undetermined.)

5.E.8. These rather brief remarks give a rough indication of some of the qualifications which must be made to all my assertions about the general applicability of the
techniques for determining truth-values, and about the possibility of discovering values of rogators by examining general techniques without applying them. They also enable us to explain away many apparent counter examples to the so-called "Laws of Logic", as being cases where the applicability-conditions for logical rogators are not satisfied, or where arguments are taken from outside the domains of definition of rogators. (Question: does this approach have any advantages over the "ranges of significance" approach, for a theory of types? Cf. 5.E.6.c.)

This concludes my account of the workings of logical constants, and also my account of the general connection between meaning and truth. We may now proceed to discuss meaning and necessary truth, and in particular to distinguish various ways in which relations between arguments may determine the values of rogators.