APPENDICES

I. Singular referring expressions 335
II. Confusions of formal logicians 340
III. Implicit knowledge 357
IV. Philosophical analysis 372
V. Further examples 381
VI. Apriori knowledge 386
Appendix I

SINGULAR REFERRING EXPRESSIONS

The discussion in the thesis has hardly been concerned at all with singular referring expressions, that is, proper names, definite descriptions and other words or pronouns which, in one way or another, refer to actual particular objects. The main reason is that in order to illustrate the existence of synthetic necessary truths it is enough to discuss sentences containing words which do not refer to anything besides universals.

There is another reason why it is most helpful to define the analytic-synthetic and necessary-contingent distinctions in such a way as not to apply to propositions which mention particulars, namely to avoid difficulties with such propositions as the following:

i) The Queen of England is a woman.

ii) I am speaking now.

iii) John's uncle is a man.

iv) He is a male.

v) Socrates is human. (Searle, D.Phil. thesis, pp. 132-7.)

The difficulty is that in each case there are reasons for regarding the proposition as analytic and reasons for regarding it as synthetic. The reason for regarding each of them as analytic is that its negation looks self-contradictory in some sense. "Queen" surely means "female monarch", so the Queen of England could not fail to be a woman. (Etc.) The reason for regarding each of them as synthetic is that it presupposes the existence of some particular, which it mentions, and so one cannot discover
that it is true merely by investigating the meanings of the words and the logical form: one must know that the particular object exists, as the result of some empirical enquiry.

The same sort of difficulty arises when we consider inferences to propositions mentioning particulars. For example, from (a) "If anyone is an uncle then he is a man" we may apparently infer (b) "If Tom is an uncle then he is a man", and the inference looks as if it is logically valid: certainly it is normally regarded as valid. But then, if (a) is analytic, why should (b) not be analytic too, since anything logically implied by an analytic proposition is itself analytic? One way out would be to say that the inference is not purely logical, since (b) presupposes something which (a) does not, namely that there is a person referred to by the word "Tom", and only one, and this cannot be guaranteed by logic.

Now consider the inference from (b) to (c) "John's uncle, Tom, is a man". Again, the inference looks as if it is logically valid, and is normally treated as if it were, but, as before, the conclusion presupposes something which is not presupposed by the premiss, namely that there is exactly one person who is referred to by "John", that he has exactly one uncle, and that that uncle is named "Tom".

Inferences of this sort are normally regarded as valid because we take their presuppositions for granted, and when this can be done logic suffices to ensure that the conclusion will be true if the premiss is. Nevertheless, the fact that there is the additional presupposition in each case seems to provide a good reason for saying that the conclusion does not state a necessary truth, and is therefore not analytic.
Whether we describe statements like (b) or (c) as "analytic" or not does not matter very much. What is usually of philosophical interest is whether statements of the form of (a) are analytic, or necessarily true. In other cases we can talk about the connections between the meanings of words as being analytic, or, in the terminology of chapter six, identifying relations, without asserting that the propositions which they express are analytic.

There is a further reason for withholding the necessary-contingent distinction from statements mentioning particular objects. Necessity is defined, in chapter seven, in terms of what would be the case in all possible states of affairs; a statement is necessarily true if no exceptions are "allowed as possible". Thus, in order to ask whether the statement "This round box is not square" was necessarily true, we should have to ask whether it would be true in all possible states of affairs, or whether one of them would provide an exception. But the normal criteria for identity of material objects simply do not extend far enough to enable us to decide, in all possible states of affairs, which box, if any, was the same box as the one referred to in the statement in question. Hence we cannot decide in every possible state of affairs whether anything is "this box", and so cannot decide whether the statement would be true in all possible states of affairs. (Indeed, in some possible states of affairs there would be nothing which was "this round box".)

The criteria for identity work up to a point, but only if we try to identify objects in different possible states of affairs which are not too different: there must be a
certain amount in common between them. This explains why we can intelligibly say: "This box might have been red", "If that wall had been painted yesterday it would have been dry by now", and so on. We can talk like this because we do not imagine other things to be very different, which is why criteria for identity work. But they could break down. Suppose the house had been bombed twenty years ago, and the wall rebuilt as before. Would it still have been this wall? And if it had been built of a different material? Or with a different thickness? Or in a slightly different position? ...

Of course, we can, if we wish, treat singular statements such as "This round box is not square" as necessary truths, simply because they are substitution-instances of universal statements which are necessarily true. This is the fact which underlies our use of such idioms as "Tom's uncle has to be a man", "If the box is square then it must be rectangular". More commonly, perhaps, the phrase "not necessarily" is used to deny the existence of a universal statement which is necessarily true of which some statement would be a substitution instance: "Her aunt is not necessarily a spinster". But I shall not apply the necessary-contingent distinction to statements mentioning particulars, for the reasons already given. This does not mean that singular statements are contingent, but that the question simply will not arise. It is of no interest for our purposes, as all interesting questions about necessity can be reduced to questions about necessary connections between universals. (See 7.B.)

It should be noted that I have defined "necessary" in such a way that it cannot always be applied to pro-
positions which mention universals. Thus, the statement "Roundness is incompatible with squareness" is neither necessary nor contingent according to my definition, since it is not clear what could count as an exception to it, unless it means "Nothing is both round and square". The definition of "necessary" in chapter seven is concerned only with what would be the case in all possible states of this world (any world in which the same universals exist as in our world). We can then ask whether in some possible state of affairs two properties would exist in the same object, and if the answer is in the negative, then the properties are incompatible. But there is no further intelligible question whether in some possible state of affairs the answer would be positive! Hence there is no clear sense to the question whether the properties would be compatible in some possible state of affairs. We may be able to invent some sense for the question whether there might have been a world in which these properties were not incompatible, but it would certainly not be a question about all possible states of this world. This illustrates the general fact that there is no question of truths being necessarily necessary or only contingently necessary, in my terminology.
Appendix II

CONFUSIONS OF FORMAL LOGICIANS

II.1. In chapter five I tried to illustrate a way of studying logic by explaining the logical properties of propositions and relations between propositions. In this appendix I wish to show how people may be led astray as a result of a concentration on methods of classifying and describing propositions according to their logical properties and relations to one another. I shall exaggerate some features of this "formal" approach to logic, in the interests of brevity and clarity: it should not be regarded as an accurate historical survey. There is probably no philosopher who has consistently made all the mistakes which I shall describe.

II.2. Many philosophers have thought that logic consists in the study of propositions which are true in virtue of their logical form and inferences which are valid in virtue of their logical form. (See, for example, p. 10 of Russell's "The Principles of Mathematics".) They have not often been quite clear about the distinguishing characteristics of logically true propositions and logically valid inferences, but they do seem to mean to refer to what I call "formal truths" and "formally valid inferences". (See 5.0.6, 5.0.8.)

The first step in such a study seems to be to describe and classify propositions according to their logical properties, or, more commonly, to classify inferences. As shown in chapter five, a convenient way of classifying them according to their logical form is to replace all non-
logical words in the sentences concerned by variable-letters. For example, "All P's are Q's" represents the logical form of "All horses are animals", and the logical form of the inference from "All stallions are horses and all horses are animals" to "All stallions are animals" is represented by "All P's are Q's and all Q's are R's = \textit{ergo} = All P's are R's", or some such symbol. In this way all propositions or inferences with the same logical form may be represented by the same symbol, and the geometrical or typographical properties of the symbol are supposed to show or represent the logical properties and relations of the propositions and inferences represented by the symbols. (Instead of such symbols, we could, of course, use such words as "disjunctive", "conditional", "universal affirmative", etc., to describe logical forms for purposes of classification.)

II.3. For a long time logicians apparently did little more than give these various forms (or at least the ones they had bothered to symbolize) names, and list those which corresponded to logically valid inferences, or logically true statements, and gradually they came to think they could forget about the propositions and inferences with which they had started and concentrate entirely on the \textit{symbols} representing their logical forms. They even went so far as to mistake symbols for propositions, with dire consequences, as will be seen.

II.4. Eventually came the discovery that a system of symbols representing logical forms could be regarded as something like an algebraic system, as follows. A few of the (simpler) symbols representing logically true
propositions could be taken as "axioms", and a few, or perhaps even only one, of the symbols representing logically valid inferences could be interpreted as expressing a rule for deriving new symbols from given ones, in such a way that the class of "theorems", that is symbols derivable from the axioms by successive applications of the rule(s) of derivation, constituted the class of symbols representing propositions true in virtue of their logical form.

The discovery of such "formal systems" amounted to the discovery of a recursive method of characterizing a class of symbols which represent propositions true in virtue of their logical form: for the axioms and rules of derivation provide a recursive definition for the predicate "is a theorem".

II.5. Extensive mathematical investigations were carried out of the various ways of characterizing such recursive systems and comparisons were made between different methods of defining the same class of "theorems", and between quite different systems, many entertaining mathematical results being obtained.

Unfortunately, some people mistook this mathematical study for a philosophical one: thus, to show that some symbol was a theorem in a formal system was thought of as a sufficient explanation of the fact that propositions with the logical form represented by that symbol were true. For example, Waismann, in his article "Analytic-Synthetic" in *Analysis*, December 1949 (pp. 31, 33, 36), implied that correspondence with a theorem of 'Principia Mathematica' or some other text-book of logic was a necessary and sufficient condition for being a logical truth.
Sometimes the construction of derivations of theorems in such a system was thought of as constituting a proof of the propositions whose forms were represented by those theorems. Logicians apparently failed to notice that any class of symbols can be represented by a suitably chosen formal system (though in some cases there may be no rules of derivation and all theorems may have to be taken as axioms), and so they thought that there was something significant about the fact that logical truths could be represented by such a system. (There is something significant, namely that our rules for the use of logical words enable more and more complex sentences to be built up using those words, their meanings being determined by the way they are constructed. But by this time philosophers had forgotten about meanings, having been mesmerized by symbols.)

II.6. The trouble was that the possibility of representing properties of propositions or relations between propositions by symbols in a formal system, together with the geometrical resemblance between such symbols and sentences in a language, led philosophers to think of such a formal system as a kind of language, or even as constituting a part of our own language. At any rate, those parts of symbols which looked like or represented our logical words, such as "and", "or", "not", etc., were identified with our logical words. A formal system was somehow thought of as providing a framework for our language, in which non-logical words or concepts could be embedded. (This attitude is clearly expressed in the writings of Carnap.) Sometimes it was hinted that insofar as any language fell short of this ideal it must be deficient.
II.7. Thus, a language came to be thought of as made up of a set of logical words, whose functions were fully defined by the rules of some formal system, together with extralogical words governed by rules of some other kind. But, since the rules of a formal system are concerned only with the symbols in that system and their geometrical relationships, since they mention nothing extralinguistic, it looked as if the rules for the use of logical words must themselves mention no non-linguistic entities. Thus it was claimed that the rules were purely syntactical. Similarly, since the property of being a theorem in the system was a recursively defined syntactical property of symbols, and since theorems represented logically true propositions, it was claimed that logical properties were purely syntactical properties of sentences. Thus it was thought that logic could be reduced to syntax. I have already argued against this in section 5.A: if logical constants were governed by purely syntactical rules, then they could never have any essential function in sentences expressing contingent truths or falsehoods.

II. 8. If formal systems provide frameworks for languages, and if formal systems have axioms and rules of inference, then surely languages must have them too? So languages came to be talked of as "systems" with axioms and rules of inference of their own. (I have argued that these concepts do not intelligibly apply to real languages, in "Rules of Inference or Suppressed Premisses?" which should appear in Mind soon.) Since different formal systems may have different sets of axioms and rules, and even different classes of theorems, it seemed that languages too might have different systems of logic, and so people quite
happily talked about "the logic" of a given language, or talked of logical truth as in some way relative to a system. (See M. Bunge, in Mind, 1961.)

II.9. The discovery of formal systems, and the possibility of giving recursive characterizations of the class of symbols representing propositions in the system (by "formation rules") in addition to the possibility of recursively characterizing the class of "theorems", sparked off a number of reductive programmes. It was found that the set of logical constants employed in a formal system could be decreased in size without effectively diminishing the number of theorems, since all theorems containing other logical constants than those chosen as primitive could be reintroduced merely by giving "definitions" for the eliminated constants in terms of the constants taken as primitive, and substituting "synonymous" symbols.

Thus the class of theorems and the class of logical constants were "reduced" to subclasses by definitional elimination, and it was thought that a similar procedure could be employed for reducing the concepts and theorems of arithmetic to those of logic. Since logic was thought of by some as merely a matter of syntax, and since it was thought that arithmetic could be reduced to logic, it seemed that arithmetic should be thought of as a syntactical science concerned with the manipulation of symbols in a formal system. Frege brought powerful arguments to bear against such views, but it is not quite clear whether he realized that they could be used against Formalist philosophies of logic as effectively as against Formalist philosophies of mathematics.

II.10. One of the very strong motivations behind the search for formal systems was the desire to find some absolutely rigorous and explicit method of proof:
hardheaded philosophers did not like to talk about "self-evidence". The discovery of formal systems made it look as if the notion of logical proof or justification could be "reduced" to formal derivation from a fixed set of axioms and definitions by means of predetermined rules of inference. The production of proof-sequences and proofs of theorems in formal logic came to be thought of as the paradigm of rigorous argument. Only deduction in a logical calculus could be regarded as strictly valid reasoning.

II.10. Eventually, some thought that the only way of proving anything was by producing a sequence of statements starting from axioms and definitions and proceeding according to the rules of inference which were supposed to be among the rules of our language. (See II.8, above.) No notice was taken of the fact that there must be some other way of being justified in accepting something as logically true, or as logically following from something else: the question "What right have we to accept the axioms or to follow the rules of inference? How do we know that we shall not be led to affirm false statements?" was not given a proper examination. Thus people tried to "analyse" the actual processes of reasoning which we follow when we are not involved in logical investigations by looking for "suppressed premises" and "rules of inference". They regarded informal proofs, such as the proofs employing diagrams which are used in geometry, as somehow inferior or inadequate: such proofs had to be replaced by something formalized. (Cf. 7.D.ff.)

The logical conclusion of this line of thought is that unless one has worked through a proof of some formal system
such as "Principia Mathematica", one is really not fully justified in believing that three plus two equals five, or in believing that if all wooden boxes are red, and not all wooden boxes are round, then not all red boxes are round, or in believing that nothing can be round and square at the same time. But people are justified in believing these and other things, and completely justified, without having gone through formalized proofs, and this shows once again that there must be some other kind of justification than that given by a formal proof. (I have tried to describe this other justification in section 7.D on informal proofs.) But perhaps, if we have this other kind of justification, then formalized proofs may be superfluous?

II.10b. Even if proof-sequences givesome sort of justification for acceptance of statements corresponding to the formulae terminating them, this justification could not be complete. The point of asserting an arithmetical or logically true proposition cannot simply be to announce that the sentence expressing it is derivable in a formal system from other symbols according to fixed rules. For then asserting such a proposition would be analogous to displaying a chess-board with the pieces arranged in a certain way in order to announce that they are in a position obtainable from the "starting position" via moves in accord with the rules of the game. If asserting the proposition has some further point, then a justification is required for assuming that mere derivability in some system guarantees that it is true, if asserted with this further point: a justification is required for regarding the axioms as true and for regarding the rules of derivation as truth-preserving. (It is forgotten that we knew how
to select logical truths and logically valid inferences before we constructed formal systems of symbols representing them.)

All that a formalized proof can do, is "show" that a sentence has a certain geometrical or syntactical structure, or that its structure is related to other structures in some way. But this is not enough for logic: in addition one must know that when sentences have certain structures, or when structures are related in certain ways, then the propositions expressed by those sentences have certain logical properties or stand in logical relations, and one can learn this only by taking account of the roles in the language of various aspects or parts of sentences, not by looking at formal systems.

II.10c. I do not wish to imply that the construction of a formalized proof can never serve any useful purpose. It can be used to demonstrate that certain syntactical relations (i.e. geometrical relations) hold between sentences expressing certain propositions. That is to say, a formalized proof rigorously, but informally, proves a theorem of combinatorial geometry.

A formalized proof can demonstrate that if logical relations correspond to certain syntactical relations, then those logical relations hold between certain propositions. (It does not show that logical relations correspond to these syntactical relations, nor does it show why they do.) In addition, just as formulae in a calculus can be thought of as providing a useful symbolic representation or "map" of propositions in a language (cf. II.2 and II.4), so can proof-sequences, which pick out certain sorts of "routes" along such a map, usefully
represent patterns of formally valid arguments, for purposes of classification or summary, for example. A formal proof can be a useful guide when one is trying to understand an argument to see whether it is valid or not, but the proof does not make understanding superfluous.

I am not trying to show that such proofs are quite useless so much as to draw attention to some mistaken views as to the purposes which they can serve, and to show how they are connected with other mistakes which arise out of a concentration on the symbols which represent forms of logical truth or logically valid inferences. The argument has been aptly summarized by Frege; (in "Translations" p. 201):

"Apparently we are being tacitly referred to our knowledge of meaningful arithmetic. But if we have a knowledge of meaningful arithmetic, we have no need of formal arithmetic."

Replace the word "arithmetic" by "logic" in this statement, and it will serve as a summary of all my remarks.

II.10d. All this seems to show that there is something fundamentally misguided in the attempt to produce absolutely explicit proofs, except perhaps as an exercise in a branch of mathematics - combinatorial geometry. It shows that there is something wrong with the Leibnizian dream of an ideal language which somehow has its meaning written on its face, so that one can settle questions about truth and falsity by mechanical manipulations alone. It is misguided because, no matter how much is written on the face of a symbol, there will always be something left out: an explanation of what the "writing" means, a description of its function in the language. Semantics cannot be reduced to syntax.
(This does not mean, of course, that there is no such thing as a rigorous proof. It merely means that one kind of analysis of rigour is wrong. See chapter seven, section D.)

II.11. One of the sources of an oversimplified view of logic (logic = syntax) is the selection of a class of "canonical" forms for study. It is obvious that if all possible symbols corresponding to the logical forms of propositions and inferences were constructed in the usual manner by replacing non-logical words in sentences by variable-letters, then many more different sorts of symbols would be obtained than have ever been encompassed within the class of symbolic forms discussed in any one text-book of logic. For example, the following are not usually listed separately by logicians:

1. All A's are B's.
2. Every A is a B.
3. If a thing is an A then it is a B.
4. Only B's are A's.

Instead, they represent the whole lot by one symbol, such as

5. Whatever x may be, if x is an A then x is a B,
which is then described as a "canonical" form. (This is similar to the old mistake of thinking that logicians need consider only propositions in subject-predicate form.)

II.11a. Normally the selection of canonical forms is done as a matter of course, following a philosophical tradition originating with some philosopher's (understandably) limited survey. But sometimes an attempt is made to justify the failure to discuss statements or inferences
not in canonical form. Several different sorts of reasons may be offered.

i) It is obviously more convenient to classify only a small class of logical forms than to take all varieties into account.

ii) The omission of forms like (1) – (4) above may be defended by the assertion that any proposition with one of those forms "obviously" means the same as a proposition with the canonical form (5), and may therefore be replaced by it without any loss of generality. (See for example, Quine's remark in "Word and Object", p. 228: "... Such a canonical idiom can be abstracted and then adhered to in the statement of one's scientific theory. The doctrine is that all traits of reality worthy of the name can be set down in an idiom of this austere form if in any idiom.") It may be added that statements in canonical form are clear and precise, whereas other statements are vague, ambiguous, or unclear. Moreover, if anyone wishes to use one of the other forms with a clear meaning he may do so by redefining it in terms of the symbols and constructions employed in the canonical forms. (Cf. "Word and Object", p. 188)

iii) Finally, it seems to be thought, sometimes, that certain canonical forms are most suitable for representing logical properties of propositions. For, if written in these forms the sentences may more effectively "show" their logical properties and relations. ("... the inner connection becomes obvious ...": Wittgenstein, "Tractatus" 5.1311.)

II.11b. There is certainly no reason why, if we find it convenient, and if only some facts of logic interest us, we should not select only a subclass of the whole class of
logical forms for purposes of study and systematic representation in symbols. But the choice of such canonical forms must always be, at least to some extent, arbitrary, depending on such subjective factors as what interests us, or what we find "obvious" and therefore not worth recording in our symbolism.

Why should we regard the following as different forms, whose logical equivalence is worth recording,

(5) Whatever x may be, if x is an A then x is a B,
(6) There is no x such that x is an A and x is not a B,

while the equivalence between (5) and, for example,

(4) Only B's are A's,
is regarded as "trivial" or "obvious", or "merely linguistic", or "merely a matter of meaning"?

Surely the equivalence between (5) and (4) is as much in need of study and explanation as the equivalence between (5) and (6)? What should we say if someone turned up who claimed to find the latter "trivial, obvious, and merely a matter of meaning" while the former equivalence was tremendously important for him? If some proposition of the form (5) turns out to be true in virtue of its logical form (e.g. if the same predicate is substituted for both "A" and "B") then why should this be any more interesting or important than the fact that the corresponding proposition of the form (4) is a logical truth?

II.11c. What I am driving at is this: if any explanation of the logical truth of, or logical relations between, propositions makes use of the fact that they are expressed by sentences in canonical form, then the explanation points to something inessential, for logical properties and relations of the same kind are found amongst propositions
expressed by other sentences. If there is some notation which renders certain logical connections perspicuous, a symbolism in which the logical properties and relations of propositions "show" themselves any more obviously than they do in other notations, then this is merely an interesting fact about that notation and its effect on us, and does not reveal any general truth about the logical properties of propositions.

After all, propositions expressed in this notation do not as obviously "show" their logical connections with propositions expressed in other notations, nor the connections between those other propositions. This is because the function of a sign is not generally shown by that sign, though if we know the functions, then, in some cases, we may more easily be able to see the consequences of their having these functions than in other cases.

II.11d. Whether propositions are expressed by sentences in canonical form or not, their logical properties and relations are due to the fact that they are built up in certain ways with logical words and constructions which have been given functions of the sort described in section 5.B. The connection between logical properties and relations of propositions and geometrical or syntactical properties and relations of sentences is a consequence of the fact that the logical constants have the functions which they do have in determining the conditions in which propositions are true or false. It may help someone to see that a proposition has certain logical properties or stands in certain logical relations to other propositions by "rewriting" it in canonical form, but pointing to the geometrical features of the new sentence does not explain
the logical properties of the proposition expressed by the old one: that has to be done by talking about their functions.

II.11s. It is a failure to see this that sometimes leads philosophers to talk about the "real" logical form of a proposition as apposed to its apparent logical form, shown by the grammatical form of a sentence. But the logical form of a proposition is the way in which its truth-conditions are determined by the meanings of the non-logical words, and that must be quite correctly shown by the sentence itself, for otherwise we could not understand it properly, and we do understand our ordinary sentences whether they are in canonical form or not. The reason why we find some types of sentences misleading is that we are philosophers who have swallowed a short-sighted traditional philosophical doctrine and fail to see a counter-example to that doctrine for what it is. (Or if we are not philosophers, then we find certain forms misleading only because we fail to think clearly and allow ourselves to be convinced that because an analogy or comparison works in some cases it must work in all.)

II.11f. Finally, in connection with canonical forms it should be noted that the tendency to regard formalized proofs as having some kind of exalted status is quite analogous to the tendency to regard some forms of propositions as somehow "superior" from the logical point of view. We think of these special kinds of proofs, or deductions, as having a "canonical form" in which logical relations are most efficiently demonstrated. Having noticed that these proofs convince us, we fail to ask why
they should do so, or why they alone should do so.

II.12. I have tried, in this Appendix, to carry out a very brief survey of some of the mistakes and confusions which arise when philosophers restrict their attention to the forms of sentences and neglect the functions of words and constructions. Partial description is mistaken for complete explanation, largely because a formal system, which is a device for representing certain features of propositions, is thought of as containing propositions, owing to the physical resemblance between its formulae and sentences in a language. The study of methods of representing facts of logic, and classifying them, leads to a mathematical study of various methods of recursively defining a class of combinations of symbols, and this study, which is really a branch of Geometry, is mistaken for a philosophical study of logic or truth or inference. The concepts invented for the purposes of such mathematical studies are mistakenly assumed to have some philosophical application: geometrical concepts referring to shapes of symbols and their interrelations are employed by philosophers when they should be talking about the functions of symbols and their interrelations. (Analogous mistakes are sometimes made by physicists, when they assume that concepts which apply only to mathematical models also have application to the reality which these models are supposed to represent. But such mistakes are less frequent because there is, fortunately, no physical resemblance between the symbols used by mathematicians and the things which physicists take them to represent.)

I think that I have been drawing attention to some of
the facts which led Wittgenstein to complain:

'Mathematical logic' has completely deformed the thinking of mathematicians and of philosophers, by setting up a superficial interpretation of the forms of our everyday language as an analysis of the structures of facts. Of course, in this it has only continued to build on the Aristotelian logic. ("Remarks on the Foundations of Mathematics", IV-48.)

The discussion of this appendix and chapter five (especially sections 5.A and 5.B) may be construed as an attempt to sort out the geometrical from the philosophical questions.
Appendix III

IMPLICIT KNOWLEDGE

III.1. Throughout the thesis I have been making remarks about things which must be known by persons who use words to make statements. But I have often qualified them by saying that such knowledge need not be explicit. In this Appendix I wish to describe some examples of what I call "implicit" knowledge, and explain why it is possible to talk about knowledge in such cases. I shall not be able to deal with the subject thoroughly or systematically, and will content myself with a few disorganized remarks. It is important to clarify the notion of implicit knowledge if we are to be clear about philosophical analysis and the nature of analytic propositions.

III.2. First I shall give a list of examples of the sort of thing I mean to talk about.

(a) In his article "Philosophical Discoveries" (in Mind, April 1960) Hare talked about some persons who all know how to do a certain kind of dance but are unable to be sure about the correct description of the way the dance goes until they actually try to do it, and he compares this with knowing what a word or expression means without being able to say what it means or how it is used.

(b) Another example is provided by a person who wishes to mention the fact that he has recently seen someone, but cannot, for the moment, recall his name. He may say: "Of course I know it - it's on the tip of my tongue."

(c) I know a tune very well, and can recognize it as
soon as I hear it, but try as I will, I cannot, for the moment, sing it or even imagine how it goes. (But if you sing the first two bars, I may be able to carry on from there.)

(d) I know a tune and can recognize it on hearing it, but if someone writes it out I may not be sure whether he has written it out correctly, until I play what he has written on the piano.

(e) I am familiar with a face, or a style of painting or musical composition, yet quite unable to say how I recognize it. I cannot say what it is about the face, or style, in virtue of which I recognize it and distinguish it from others. Even when confronted with the face, or an example of the style, I may be unable to describe the distinguishing characteristics.

(f) A person who can type very easily, even when blindfolded, may find it very difficult to describe from memory the relative positions of the keys on the typewriter.

(g) A person tries to describe everything in a room he has just left, and is sure he has left out nothing. Then someone asks: "Was there a carpet?" He replies: "How silly of me! Of course there was a carpet, and I knew that very well. I don't know why I didn't think of it."

(h) We can all count, and can tell, given any numeral written out in English or in Arabic notation, which is the next one in the series. But most persons who can do this cannot give a general formulation of the principle for going from one to the next, despite their ability to apply the principle. (Cf. 6.E.5, 6.E.6.) Even if someone else offers a formulation, they may not be able to think clearly
enough to tell whether it is correct or not.

III.2.a. This should be compared with some of the following facts mentioned in the thesis.

1) I asserted that talk about meanings presupposes the existence of criteria for identity of meanings, at all levels, in section 2.B; then, in 2.B(note), I allowed that people who talk about meanings need not explicitly know which criteria they are relying on.

2) In chapters three and four I described various kinds of correlations between words and properties which explain how we use descriptive words. But one need not be able to formulate explicitly the principle on which one decides whether to call objects "horses" or not. One may use a word according to a complicated procedure, and yet not know in an explicit way what that procedure is. (Cf. 3.D.9.) One may use a word according to several different rules superimposed in an indeterminate way, without realizing this until (e.g.) one starts thinking about difficult borderline cases. (Cf. 3.E.2, 7.D.11. note, and 4.B.2.)

3) In 5.A.3 and 5.A.11 I described techniques which we have to learn to use for discovering whether statements using the words "is", "or" and "all" are true or not. A person must know what the technique is in order to understand sentences using the words; but he need not know in an explicit way, for he may be unable to distinguish between correct and incorrect formulations of the rules for the words. The techniques may be learnt by example and memorized, without any explicit description ever being formulated by pupil or teacher. (5.A.6, 5.B.8, 5.C.9.)
III.3. Each of these examples is puzzling. In each case we want to say that there is something a person knows all the time, or really knows, despite his inability to give a correct answer to a question about it. He knows what he is doing, that something is the case, how to do something, what something is, etc., and yet, without deliberately deceiving, gives the impression of not knowing. What do we mean by saying that he really knows? What explains his inability to answer correctly in these cases?

I believe that the answer to these questions is given by the fact that there are a great many different tests for knowing any one thing, and passing any one of them counts as a sufficient justification for the claim to know, or the assertion that someone knows, provided that there is no reason to think that success in the test can be explained as an accident or some kind of lucky guess.

The fact that I can type correctly without looking justifies my claim to know the relative positions of the keys on a typewriter, despite the mistakes in my attempted description of their positions. I know where my pen is because, as soon as I need it, I go straight to the right place, despite the fact that if someone had asked me where it was I might not have been able to answer correctly. I know how the features on a face are arranged because I can recognize the face and distinguish it from other faces despite my inability to describe the peculiarities in virtue of which I recognize it. I know what the technique is for deciding whether a statement of the form "x is P" is true or not, despite my inability to formulate the technique, since, when confronted with such statements, and told the meanings of the non-logical words, I am able to decide whether they are true or not.
III.4. There are many different tests for a person's knowing any one thing, and his passing any one of them counts as strong evidence that he knows. But there is no one of them that he must pass in order to show that he knows: when he fails one of the tests this need not count as strong evidence for his not knowing, since it may often be assumed that there is an explanation for his not passing the test. (E.g. for his giving the wrong answer or not being able to think of the right answer.) Some of these explanations of an apparent lack of knowledge are the following:

First of all there is a whole family of cases which need not be discussed in detail. A person may quite sincerely give a wrong answer despite his knowledge of the correct one, simply because of a slip of the tongue, or on account of his being absentminded, or preoccupied with something else, or because he has misheard the question, or because giving the correct answer requires concentration and he has a headache. All these are cases of temporary muddle or confusion, and can usually be detected by asking the person again, in a suitable tone of voice!

III.4.a. Next there is the relatively uninteresting case where the person is unable to express his knowledge in words simply because he does not know any words which could express it adequately, either because he hasn't learnt any or because he cannot think of them at the moment. A person who knows the difference between the sound of a clarinet and the sound of a flute may simply not think of saying that the former has more upper harmonics, or that the latter is "purer" or "more naive", or "less reedy".

Connected with this is the case where a person is
not able to give the correct answer to a question simply because he has not thought of that answer as a possible one. But as soon as it is suggested he recognizes it as correct, and if several are suggested he can pick out the correct one from among them. His passing this test (perhaps in addition to his displaying his knowledge in his behaviour) shows that he "really knows", despite his failing the more difficult test of having to think up the right answer for himself. (Compare: in philosophy the difficulty often lies in thinking up the correct answer, not in seeing that it is correct, once stated.) (Compare example (g) in III.2.)

III.4.b. In some cases, a person may be unsure whether a description is correct or not, not because he has not noticed the possibility, but because it describes from a new point of view. I may know how to recognize a tune by its sound, and yet be unsure about recognizing it when written down, despite the fact that I can read music. (See example (d).) I may, in addition, be unable to recognize it when played backwards or upside down. I may recognize the feel of something (e.g. a familiar chair) and yet be unable to distinguish it from others by the way it looks: and for this reason I may be unsure about the correct description of the way it looks. I believe this may apply to one's knowledge of how to do something, such as a dance, or touch-typing. (See examples (a) and (f), of III.2.) A person who is able to tell whether he is doing a dance correctly or not (he knows which movements he is supposed to make, and he knows which movements he is making), may be quite unable to be sure whether other persons are doing it correctly when he watches them. Similarly, if he is given a description of the
dance from the point of view of a person watching its performance he may not be sure whether it is correct or not. He may find out by making the movements and looking to see whether they fit the suggested description or not. In that case he is observing himself in two ways at once, or, more accurately, he knows in two different ways what he is doing.

III.4.c. A slightly different case is the following: a person may know how something is done in the sense that he can do it in an unthinking way when he has to, and yet when he stops and thinks about it he may not be sure. For example, an experienced flautist may be able to play a descending chromatic scale at high speed on the flute, but if his fingers are placed in the position for one note and he is asked to indicate which must be moved up and which must be moved down for the next note down the scale, he may have to pause and think for some time, especially if he is not actually holding a flute. He may find it even more difficult to say which fingers must move if he is not first permitted to move them. This fact, that he cannot be sure about the individual steps of a routine which he knows how to apply quickly and unthinkingly, may help to explain his difficulty in telling a pupil which fingers to move and when, in addition to the factor already pointed out, namely that he may be able to recognize the correct movements from the point of view of an agent while being unsure about them from the point of view of an external observer.

III.4.d. Yet another possible explanation of a failure to give the correct answer is the fact that one may be able to apply a technique which is very complicated,
insofar as it has several stages, or insofar as exactly how it goes at any stage may depend on other things. For despite one's ability always to go on from one stage to the next correctly, even where how one goes on depends on what the technique is being applied to, one may not be able to think about all the stages, or to recall all the possible variants, when one is trying to describe the technique in the abstract. For example: a chemist who has been trained to identify samples of some substance, which can occur in several varieties, for which the tests are slightly different, may make a mistake in describing all the tests, despite the fact that he never errs in performing them. Similarly, we may find it difficult simply to sit back and describe all the observable factors which we have learnt to take into account in deciding whether an object is a horse or not (or in deciding whether a person has an intention to do something or not), despite our ability to make the decision when necessary.

In the previous case (III.4.c.) a person who could recognize correct applications of a complete routine might be in doubt about individual stages of the routine. In this new case a person who is sure about any one of the stages when asked may be in doubt if asked simply to describe them all. The complexity of the routine explains his failure.

III.5. I said that in each of these cases a person may be described as knowing something or other despite the fact that he fails some test for knowing. The reason why we do not take the failure as a criterion for his not knowing, is that we are able to explain the failure in some other way than by saying that he doesn't know. This
is supported by the fact that in most cases he may be able to pass the test later on without having gone through a process of acquiring the relevant knowledge inbetween.

For example, the person who gives the wrong answer because he is absent-minded simply has to think again. He need not learn again.

When I have a person's name "on the tip of my tongue," and correctly pick it out from among several suggestions offered to me, all I needed was to hear the name to bring it back to mind: in such a case I do not learn that that is his name.

The person who cannot describe the way a dance goes until he does the dance does not thereby learn how the dance goes. What he learns is how to describe the dance from a new point of view.

The person who cannot describe something with which he is perfectly familiar because he must first learn the appropriate vocabulary is not thereby acquiring the knowledge which he is able later on to express in his new vocabulary. Learning the name of a colour is not the same thing as learning that that is the colour of my table, even though it may enable me for the first time to say correctly what the colour of my table is.

To summarize: many different sorts of things may enable a person to know in an explicit way something which he previously knew only implicitly. We say in such cases that he nevertheless "knew" previously because the process in which he learns to express his knowledge or to pass one of the tests which he previously failed, is not the same sort of process as is required for acquiring the knowledge. (He does not make the relevant empirical observations, or
examine properties to discover their inter-relationships.) He is all the time potentially able to pass the test, and the evidence for this, apart from his later success, is the fact that he was previously able to pass some other test which displayed his knowledge (cf. III.3.). (I shall not discuss the question how passing one test counts as evidence of ability to pass others.)

III.6. All this may help to explain why I was able to describe some of the things which people know when they know how to talk, without fear of being contradicted by the fact that my descriptions would probably come as news to many people who know how to talk! These people know how to talk, but they do not know explicitly that their descriptive words have the meanings which they do have in virtue of being correlated with observable properties as described in chapters three and four; and neither do they know explicitly that to the logical form of a proposition there corresponds a technique for determining truth-values for sets of non-logical words by examining the way things happen to be in the world. Their knowledge of all this is implicit, and to say that they know it all is to say that it accurately describes what they are doing when they decide whether to use some word to describe an object, or whether a sentence expresses a true proposition, or whether two persons understand some sentence in the same way. Their not knowing explicitly may be explained by factors of the sorts described already.

But there is one sort of factor which can be very important, and which I have not yet discussed; I shall do so now.
III.7. A possible explanation of a person's not knowing that in doing A he is doing B, C, D ... may be the fact that he does not have the concepts which would enable him to think about his activity in this way. More specifically, he may not have the metalinguistic concepts which would enable him to think about and describe the way he uses words. For example, the child who can use the word "cat", but does not have the concept "word", can hardly be expected to know explicitly how it uses the word "cat". Similarly, the explanation of a person's inability to say how he uses the word "horse", for example, may not only be that the procedure for picking out horses is too complex (see III.4.d and also 3.B.5 and 4.A.6), but in addition that he does not have the concepts "meaning", "property", "semantic correlation", "disjunctive range", etc. he does not have the concepts which I have used in my description of what people learn when they learn to talk. (Similarly, philosophers have hitherto been unable to give a correct explicit account of analytic propositions, despite their implicit knowledge of what it is for a proposition to be true by definition, on account of not having something like the concept of a "rogator" [see section 5.B], or so it seems to me.) A person may acquire the concepts which enable him to know explicitly that he uses his logical words or descriptive words in a certain way, without actually being taught that he uses them in that way. He thereby learns to say what the words mean, but he does not learn what they mean - he knew that all along, since he could use and understand them.

III.7.a. Now we can see how to cope with the difficulty mentioned in 6.C.4. I defined "identifying fact about the meaning of words" to mean "fact which must be known
by anyone who knows the meanings of those words". The
difficulty was that someone might learn to use the word
"gleen" to refer to the combination of the properties
which are referred to by the English words "glossy" and
"green", without knowing explicitly that the property
referred to by "gleen" was the combination of the properties
referred to by "glossy" and "green". He might not know
the meanings of the words "glossy" and "green", and he might
not have the metalinguistic concepts which would enable
him to understand a statement about the meanings of de-
scriptive words. Still, he knows the fact in question
implicitly, since he makes use of it in employing the word
"gleen". He decides whether to describe an object as
"gleen" or not by looking to see whether it has the two
properties in question. If he decides in any other way
he does not understand the word "gleen" as I have described
it: if he does not know, even implicitly, that it is
correlated with those two properties, then he does not
know its meaning. (This is still not quite clear: a
complete discussion would require an investigation and
comparison of the following expressions: "Knowing what the
word 'W' means", "Knowing how to use the word", "Knowing
the meaning of the word", "Knowing that the word 'W' means
...", "Being able to understand the word 'W'", and so on.)

III.7.b. It is important to distinguish the acquisition
of new metalinguistic concepts, which enable one to express
one's knowledge of facts about the meanings of words, from
the acquisition of other sorts of concepts, which enable
one to discover new facts about the meanings of words,
facts which one did not know previously even though one
understood those words. Consider, for example, the
concept of a "starlike" figure, introduced in 3.D.5. A
starlike figure is one which is bounded by straight lines meeting alternately in reflex and acute angles. Now a person may be able to use the word "square" to refer to the usual recognizable property, without having the concept "starlike". I may teach him the new concept by giving a definition, or showing him examples, without mentioning squares at all. Having acquired the new concept he may then notice, for the first time, that no square is starlike. He discovers a new fact about the property referred to by the word "square", and thereby learns that the words "square" and "starlike" are incompatible descriptions. But he does not thereby learn an identifying fact about the meaning of the words, for in order to acquire this knowledge it was not enough for him to acquire new metalinguistic concepts.

Consider another example: I may know how to use the expression "Daisy-daisy" as the name of a tune, and be able to recognize the tune on hearing it, without knowing, even implicitly, that the first and second intervals of the tune are thirds (first a descending minor third, then a descending major third), or that the first three notes form a major chord, on account of not having the concept of a musical interval, or a major chord. Suppose someone teaches me to pick out musical intervals and name them, and then one day I hear the tune I knew previously, and notice immediately that the first two intervals are both thirds. Have I acquired explicit knowledge of something I knew previously in an implicit way? Surely not. It seems much more reasonable to say that I have discovered a new aspect of the tune; I had not previously noticed the possibility of looking at a tune as a sequence of musical intervals, and I in no way made use of the
possibility, implicitly or otherwise, for I could only have done so if I had had the appropriate concepts. (How can a person who is unable to tell whether two intervals are the same or not ever make use of the sameness of two intervals? It should not be forgotten that this is a phenomenological essay: I am not interested in what would be given as a causal explanation of how he recognizes the tune. See section 1.B.)

III.7.c. This difference between acquiring metalinguistic concepts which enable one to say explicitly how one had previously been using words, and acquiring other sorts of concepts which enable one to discover new facts about the meanings of one's words, or about the properties to which they refer, is one of the factors which lies behind the distinction between an identifying (or analytic) fact about meanings and a non-identifying (or synthetic) fact about meanings, of which so much use was made in chapter seven. This is what justifies talk about synthetic necessary truths (whose necessity has to be discovered by examining properties, perhaps with the aid of informal proofs - see 7.D), and shows that the term "synthetic a priori" is not just an old label with little explanatory force, as averred by Hare, on p. 145 and p. 153 of "Philosophical Discoveries".

III.8. To sum up: there are many kinds of things which one may know implicitly without being able to express the knowledge in words, or to answer questions about it. This inability may be explained in any one of a number of different ways. It may also be removed in a number of different ways, none of which involves actually acquiring
the knowledge in question: they all involve merely learning to express the knowledge in a new way. We say, in such cases, that one knows, despite the inability to express the knowledge, because one is able to use it, in applying a technique, in carrying out a routine, in making allowance for facts, etc.

I have tried to distinguish cases where implicit knowledge of how words are used is made explicit, from cases where a new discovery is made, where something new is discovered about previously familiar meanings, namely a connection with other meanings or some other previously unnoticed aspect.

(Some more remarks about implicit knowledge will be found in Appendix IV on "Philosophical Analysis". See also remark about "implicit justification" in 7.E.5.)
Appendix IV

PHILOSOPHICAL ANALYSIS

IV.1. It was suggested in 1.A.5 that clarification of the analytic-synthetic and necessary-contingent distinctions might help to solve problems about the nature of philosophical analysis, and perhaps lead to methodological advances. Not much has been said on this in the thesis, and in this appendix I shall make a few vague remarks, in the hope of suggesting lines for more detailed investigation.

IV.2. Philosophical analysis, which may also be described as "conceptual analysis", is essentially the search for identifying relations between the meanings or functions of words or sentences or types of linguistic constructions. (See section 6.C, and 6.F.7, ff.) This is what seems to lie behind such questions as:

(a) Can one be pleased without being pleased at or about anything?

(b) Is there something queer in the assertion "I have definitely decided to go, but I am sure I shall not"?

(c) Is it part of the meaning of 'table' that tables are used in certain ways, or have certain functions, or is it just an additional fact about tables?

(d) When a person asserts that something is the case, does he imply that he believes that it is the case?

In answering questions like these one is presumably drawing attention to connections between concepts or features of concepts, and this means drawing attention to identifying facts about the meanings or functions or uses of words or other expressions. From the discussion of section 2.B
and 6.F.7, ff., it should be clear that there are many different "levels" at which meanings or functions can be related, any of which can explain the existence of implications between utterances, or the queerness or oddness of certain utterances (self-contradiction, or analytic falsity, is just one sort of case). It would seem to be important to devise some principle for systematically classifying such identifying relations, if philosophical analysis is not to look like the piecemeal collection of linguistic oddities.

IV.3. For example, there are connections between the describability-conditions of descriptive words, between the techniques of verification corresponding to logical forms of sentences, between the purposes served by the utterance of statements, between the preconditions (existence of social habits or institutions or empirical regularities in our physical environment) for the efficacy of certain sorts of utterances. In all these cases there may be identifying relations at one level, or relations between one level and another. Thus, if one of the conventions of a language is that the utterance of a statement of the form "I intend to do X" primarily serves the purpose of giving people the assurance that X will be done, then identifying relations between appropriateness-conditions for this sort of utterance and truth-conditions for the utterance of statements of the form "I believe that I shall not do X" may generate a kind of queerness manifested in the utterance of "I intend to do X but I believe that I shall not do it". The queerness is to be accounted for, not by conducting empirical observations of what people can believe and intend, but by examining
the knowledge of meanings which we normally apply when we talk, just as we account for the necessary falsity of analytically false statements. (See section 6.E.) The difference is merely that in the latter case we are concerned only with truth-conditions.

I shall not now try to describe a system for classifying such identifying relations between meanings and functions and the consequences which they can have.

(It may turn out that we must also allow for the possibility of non-identifying relations between meanings, analogous to those relations between universals which were described in sections 7.C and 7.D.)

IV.4. Now it would certainly be of some interest to see what sort of system could be used for classifying various sorts of connections between meanings, and the consequences of such connections. This would amount first of all to an extension of the Aristotelian classification of forms of inference, of far greater significance than the mere extension to take account of more varieties of logically valid inference (the much-vaulted achievement of modern symbolic logic), and secondly to an extension of my explanation (in chapters five and six) of the existence of logical relations and properties of propositions. But this interest in principles of classification can certainly not explain why philosophers should debate with such great interest questions of the kind illustrated in IV.2. (System-building is, after all, supposed to be out of fashion.) Why are they interested in finding out whether it is actually part of the meaning of "table" that tables have certain functions, and whether the English words "red" and "green" are actually analytically incompatible, instead of merely noting that there are these possible linguistic
conventions, and that adopting them would have certain consequences?

IV.5. Perhaps some philosophers are simply interested in empirical questions about how people use words, and these questions are not entirely trivial, despite the fact that we may already know how the words are used. For our knowledge may be implicit, and, as pointed out in the previous appendix, there may be some difficulty in expressing such knowledge explicitly, especially if superficial analogies lead us mistakenly to expect that the answers will be of a certain kind. (Cf. App. II.11.e, above.) But the best way to serve this sort of interest in empirical questions is to carry out empirical surveys (e.g. using statistical methods), and it would be important to allow for the possibility that in general there will be no definite answers to such questions, since how a word is understood may vary from person to person, and even an individual may understand it in an indefinite way. (See the discussion of indeterminateness in chapter four, and also 3.E.2, 5.E.7.a, 6.D.3, 7.D.11.(note).)

IV.6. However, most of those who indulge in conceptual analysis are not inclined to use the methods of popularity pollsters, and this is not merely a matter of laziness: they are not really interested in how people actually talk, though their words often seem to belie this. What else are they trying to do then? What sorts of non-empirical questions can they be trying to answer?

One kind of non-empirical question is the question whether a philosophical distinction is vacuous, or whether a philosophical system of classification works in this way
or that, or not at all. Thus, a philosopher (Kant) who has begun to describe a system of classification of the sort envisaged above, might try to illustrate its application by assigning particular examples to their place in it, and his opponents might dispute that his principles of classification have been correctly applied in these particular cases. He says the statement $S$ has certain features in virtue of which it satisfies the conditions which he has laid down for being synthetic. They say that it does not. But now there comes a confusion between the question whether $S$ has those features as it is in fact understood, the question whether having those features entails satisfaction of the conditions for being synthetic and the question whether it is possible for any statement to satisfy those conditions. Compare the following case. A mathematician tries to prove that any geometrical figure which has the property $P$ also has the property $Q$, and he does so by drawing a diagram with construction-lines, etc. (See section 7.D.) Now if the case is sufficiently complex there may be a debate as to whether the figure which he has drawn actually has the property $P$, or whether it actually has the property $Q$, or whether the construction-lines as he has drawn them actually serve the purpose they are meant to serve. These (semi-empirical) questions about the particular diagram may then be confused with other questions about properties, such as whether they are necessarily connected, or whether it is possible for objects to possess them at all, etc.

Thus, what really underlies an interest in an apparently empirical question about how words are actually used is an interest in a non-empirical question as to whether and how a system of classifying possible ways of using words may be applied. But the failure to distinguish
the empirical from the non-empirical questions may lead philosophers into endless disputes about "What we really mean" — endless because what we really mean is too indeterminate for either side of the dispute to be correct. (This seems to me to be clearly illustrated by disputes as to whether "I know I am in pain" is odd, disputes as to the connection between expressions of intentions and predictions of one's future actions, and disputes as to whether it is part of the meaning of "good" that certain types of men are good men or whether it is part of the meaning of "good" that believing certain types of men to be good men is connected with being inclined to behave in certain ways. In the last case, not only are empirical and non-empirical disputes confused, but, in addition, practical disputes about how we ought to use the word "good" are mixed in too.)

The existence of this sort of confusion is what led me, in 2.C.10 and 7.C.8, to stress the fact that even if it is established that in English the statement "All triangles have three sides" is analytic, this does not close the question whether it is possible to use the word "triangle" to refer to the property of having three angles in such a way as to make the sentence "All triangles have three sides" express a synthetic necessary truth. We were not concerned with the question whether some statement in English is actually synthetic, but with the question whether it is possible for certain sorts of sentences to express synthetic propositions.

IV.7. We have so far found that conceptual analysis can serve the following purposes: 1) It can provide empirical reports on linguistic usage, by making our implicit knowledge of how we talk explicit, though this purpose might
be better served by conducting statistical surveys.  
2) It can be a disguised account of the workings of some system of classification of kinds of relations between meanings and functions of linguistic items, and their consequences.  
3) The discussion of the way in which particular words are actually used may serve to illustrate or clarify or provide counter-examples to general statements about systems of classification (or about possible ways of using words) in much the same way as the particular diagram used in an informal proof can enable one to see the truth of some general statement about geometrical properties, or provide a counter-example.

IV.8. However, this may make it look as if philosophical analysis is concerned only with language and theories about language, but that is not so. For a conceptual analysis, in drawing attention to previously unnoticed facts about the ways in which we actually use words, serves also to draw attention to a possible way of classifying the things referred to or described by these words: types of material objects, states of mind, kinds of behaviour, etc. It may draw our attention to a system of classification with which we are all familiar in one way, since we employ it all the time; though in another way it has general features of which we are unaware, for the sorts of reasons described in the previous appendix. ("So that's what I've been doing all the time - I'd never have guessed" may be the expression of having made a philosophical discovery!)

IV.8.a. What interests the philosopher, however, is not so much the fact that we do classify things in this way or in that way, as the fact that it is possible to
classify them in one way or another. (This may be important in dispelling philosophical prejudice as to what it means for consciousness to exist, for example.) The interest in possible ways of classifying things need not be fed only by analysis of concepts which we actually employ: the scientist or philosopher may draw our attention to new possible ways of classifying things, perhaps by teaching us to use new concepts.

(A very interesting and difficult question, which underlies much of Wittgenstein's discussion in "Philosophical Investigations" is the question whether and to what extent the possibility of adopting certain systems of classification depends on what is actually the case in the world. In a more specific form, this becomes: the question whether I have been right in saying that the existence of observable properties does not depend on which particular objects actually have those properties. (See 2.D and 7.A.) Would it make sense to talk of the size or shape of objects if everything were constantly changing in size and shape? Colours? Etc.)

IV.8.b. The wish to understand the world, in a philosophical way, is, at least partly, constituted by the wish to know how things in the world may be classified. (Can we divide the world up into material entities and mental entities, or are there only material entities, some of which have certain observable properties while others look and act differently? Are there electrons and other subatomic particles out of agglomerations of which the other things in the world are formed, or must the place for electrons in our scheme of things be explained in terms of ways of classifying observable macroscopic phenomena
such as flashes on fluorescent screens and clicks in geiger counters?) The wish to know how things may be classified may be satisfied by finding out the answers to questions of the form "Can we say that ...?" "If such and such had been the case, might we have said that ...?" "Can the word so and so be applied in such and such a sort of case without changing its meaning?" Indeed, this may be the only way of making satisfactory progress. But the fact that these questions are explicitly about words disguises the fact that they are implicitly about things. This is why the criticism that so-called "linguistic philosophy" is just a dilletantish enquiry into empirical facts about linguistic conventions, merely misses the point. (This misunderstanding is excusable, however, since its practitioners often miss the point themselves. This, like the rejection of the possibility of synthetic apriori knowledge, is one of the manifestations of the neurotic fear of doing anything resembling old-style metaphysics.)

IV.9. To summarize: in addition to the purposes listed in IV.7, above, philosophical analysis can also, in an indirect way, help to provide answers to very general questions about the world of our experience, which, according to the Pocket Oxford Dictionary, is the business of philosophy.
Appendix V

FURTHER EXAMPLES

V.1. In section 7.C the notion of a non-identifying relation between the meanings of words was illustrated by means of geometrical examples. Geometrical properties are not the only ones which may stand in synthetic relations, but they give the assertion of the existence of synthetic necessary truths its strongest support. Some additional problematic examples will be mentioned now, but not discussed in any detail.

V.2. Our first examples involve colour-concepts. Discussions of the relations between colours or between colour-words can be very confused if the distinctions made in chapter three are not taken seriously. Thus, we have seen that one and the same colour-word "red" may have different sorts of meanings depending on whether it is an f-word directly correlated with a hue (see 3.A.1), a d-word disjunctively correlated with a range of specific shades of colour (see 3.B.2) a p-word correlated with a range of specific shades picked out by some procedure (see 3.D.2), or a word correlated with the disposition of normal persons to say "red" (see 3.D.2.note). The words given their meanings in these various ways may have the same extension. So our first set of problematic examples come from questions of the form: Is it analytic that everything which is red in one of these senses is red in another of these senses? (This is a question which cannot be asked in ordinary English, since the word "red" as ordinarily understood has neither one of these meanings nor another, and our
ordinary vocabulary does not make provision for distinguishing these several senses easily. See 3.5.2.)

V.2.a. Now suppose that the word "scarlet" is an f-word, referring to just one specific shade (a shade of red). Then we can ask whether the sentence "All scarlet things are red" expresses an analytic proposition. Owing to the indeterminateness of the meanings with which words are normally understood, this question probably has no answer (see section 6.D). By distinguishing various possible (sharply identified) meanings of "red" we can understand the question in such a way that it has an answer. Thus, if "red" is a d-word disjunctively correlated with a range of specific shades, of which the shade referred to by "scarlet" is one, then the sentence in question expresses an analytic proposition.

On the other hand, if "red" is an f-word simply correlated with the hue redness (3.A.1), then the meaning of "red" and the meaning of "scarlet" can probably be identified independently of each other: one could learn to recognize the specific shade without being able to see the hue, and one could learn to recognize the hue without ever having seen the specific shade. It follows that if there is a necessary connection between the hue and the shade this must be discovered by examining the two properties, in which case the proposition that every-thing which has the shade in question has the hue in question must be synthetic and necessarily true.

V.2.b. People sometimes say that it is merely analytic that nothing can be two colours at the same time. Certainly, we could adopt n-rules to ensure the incompatibility
of colour words (see section 4.C), and it may be the case that we do. But perhaps we do not need to: perhaps there are ways of understanding colour-words so that the connections between them are synthetic. Exactly what sort of relation holds between a pair of colour words will, of course, depend on the sorts of meanings they have.

Thus, if "red" and "yellow" are both d-words dis-junctively correlated with ranges of specific shades of colour (with no overlap between the ranges), then the question of the incompatibility of "red" and "yellow" is logically equivalent to the question of the incompatibility of different specific shades of colour.

On the other hand, if "red" and "yellow" are both f-words, directly correlated with hues, then their incompatibility depends not only on the impossibility of finding an object with two different specific shades of colour, but also on the impossibility of finding an object which has a shade of colour which is simultaneously a shade of red and a shade of yellow, that is an object which has two hues at once, though only one shade of colour.

If one is a d-word correlated with a range of specific shades of colour, and the other is an f-word correlated with a hue, then the incompatibility will depend on the impossibility (for example) of finding an object with one of the specific shades correlated with "red" and the hue correlated with "yellow". Is it analytic that nothing which is scarlet in shade can be yellow in hue?

If both words are p-words, correlated with properties by means of procedures, of the sort described in section 3.D, then the relation between them may be still more complex and problematic.
V.3. Examples are also forthcoming when we consider properties of sounds.

What sort of fact is it that if two musical intervals with the characteristic sounds of a fifth and a fourth are added together, then the outer interval will have the characteristic sound of an octave?

What sort of fact is it that only if the three notes of a chord are separated by two of the following intervals: a third, a minor third, and a fourth, can they form a triad with the characteristic sound of a major chord? Can all these musical properties be identified independently, or can they be identified only by specifying their relations? Or are the relations contingent?

V.4. Another example: no surface is both glossy and mat at the same time. Analytic or synthetic? Necessary or contingent?

V.5. There seem also to be relations between mechanical properties, which are really the same sorts of things as the geometrical properties already discussed, except that motion comes in too.

For example, if a rod remains straight and its midpoint remains fixed while one end moves down, then the other end moves up.

If two gear wheels are meshed, their shapes and distance apart remaining constant, then if one of them turns about its axis, and neither penetrates the other, then the second one will move too. (This is the sort of thing which enables us to predict what will happen when one part of a machine starts moving, if none of the parts bends or disintegrates or penetrates the others.) Is this analytic?
V.6. I believe that more examples can be found by considering relations between numbers. First of all it should be noted that among the several different concepts superimposed in our ordinary arithmetical concepts are "perceptible" numerical concepts. For it is possible to learn to recognize the number of objects in small discrete collections just by looking, and without counting or otherwise correlating the objects with anything else. Similarly, one might learn, by being shown examples, to recognize simple operations performed on such sets, such as addition and subtraction, nothing being allowed to come into or out of existence or merge with anything else during such an operation. Then, by examining these observable numerical properties and operations, perhaps with the aid of informal proofs of the sorts described in section 7.D, we may be able to see connections which justify the assertion of such statements as "A two-set added to another two-set with which it is disjoint yields a four-set". "A five-set can be divided into a two-set and a three-set."

It is arguable that such statements, if understood in one way, are both synthetic and necessarily true. But I shall not go into the argument.

V.7. All these examples rely on the fact that there are properties which are independently identifiable. The sort of thing that is meant by saying that such properties can be identified independently of one another was illustrated by the discussion in section 3.C of the way in which properties can explain our use of descriptive words.

In addition, the examples rely on the fact that in order to perceive the necessary truth of the statements which are alleged to be both synthetic and necessarily true, it is necessary to be acquainted with the specific kinds of properties referred to: purely logical, or topic-neutral, enquiries will not suffice.
Appendix VI

APRIORI KNOWLEDGE

VI.1. I think the arguments of chapter seven show that the common tendency to confuse the terms "analytic" and "necessary" ought to be resisted. Similarly it is tempting to confuse the terms "necessary" and "apriori". I shall now try to show briefly that this is undesirable.

VI.2. Kant asserted that one of the marks of apriori knowledge was necessity, and there is something in this. If a statement is contingently true, then it would be false in some possible state of affairs, so there are no reasons why it should be true which can be known without discovering which possible state of the world is the actual one. Hence the truth of a contingently true statement has to be ascertained by empirical observation of the facts to make sure that there are no counter-instances. Thus no statement which is not a necessary truth can be known apriori to be true, from which it follows logically that if a statement can be known apriori to be true then it is necessarily true.

But the converse does not follow. There may be statements which are necessarily true which are not known to be true at all, let alone known apriori. Some necessarily true statements may be known to be true only on the basis of empirical enquiry: a person who fails to realize that every object bounded by four plane faces must have four vertices may establish the truth of "Every object bounded by four plane faces has four vertices" by carrying out a survey of objects bounded by four plane faces and counting their vertices. Indeed, it seems likely that there
are some truths which cannot be known apriori by any human being despite their necessity, owing to the complexity of the connections between universals in virtue of which they are necessarily true. Perhaps there are kinds of connections between properties which simply cannot be discovered by examining those properties: causal connections may be like this, in which case there are necessary truths which cannot be known a priori at all. At any rate, it is clear that the concepts "necessary" and "known apriori" are distinct, though there is a connection between them.

VI.3. This helps to show that the term "apriori" should not be applied to statements or truths: it applies to kinds of knowledge or ways of knowing. I have not defined the expressions "apriori" and "empirical", and it is probable that their use in philosophical discussion is even more confused than the use of the terms "analytic" and "necessary". One way in which the aprior-empirical distinction may be applied is as follows. Where the truth-value of a statement depends on whether or not certain particular objects have certain properties or stand in certain relations, it is, in general, necessary to ascertain the truth-value by carrying out observations to see which particular objects exist, and which relations and properties they instantiate. This is an empirical way of acquiring knowledge: one observes particular contingent facts. However, as shown in the thesis, there are some cases where it is not necessary to carry out such an investigation, since its outcome can be discovered merely by examining the properties and relations concerned and perceiving connections between them. This is an apriori way of acquiring knowledge: one does not discover by empirical observation how things happen to be in the world which
might have been otherwise (see section 7.A). This is the distinction which I have had in mind whenever I used the term "apriori", namely the distinction between knowledge obtained by observation of particular facts and knowledge obtained without observation of particular facts.

VI.4. It is sometimes suggested that apriori knowledge, and knowledge of necessary truth, is not derived from or dependent on experience. But it is not at all clear what this means, unless it is an obscure version of what I have just said.

For there are two ways in which the examples of apriori knowledge of necessary truth mentioned above are derived from or at least dependent on experience. First of all, each of the necessarily true statements which I have discussed employs descriptive words referring to observable properties, and, in general, in order to have the requisite concepts and understand the statements it is necessary to have had some experience of objects with these properties, or properties similar to them. At any rate, this is how we do in fact usually acquire such concepts. Secondly, experience comes in when one perceives the connections between such properties. For example, one may have the required insight while looking at a particular diagram which exhibits the properties in question. So in this sense apriori knowledge may be based on particular experiences (rather than on detached and lofty exertions of "Pure Reason"). But it is still not empirical in the sense defined above. Whether there is knowledge of a sort which is completely independent of all experience in some sense is dubious: what could it be knowledge of?