Plans and the Computational Structure of Language

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> *Not only speech, but all skilled acts seem to involve the same problems of serial ordering, even down to the temporal coordination of muscular contractions in such a movement as reaching and grasping. Analysis of the nervous mechanisms underlying order in the more primitive acts may contribute ultimately to the solution of even the physiology of logic*.
>
> Karl Lashley 1951:122

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The Claim

- AI Planning formalisms such as the Linear Dynamic Event Calculus (LDEC) provide a transparent notation for causal/teleological knowledge representations, in which:
  - Composition **B** characterizes *seriation* of actions in plans.
  - Type-raising **T** characterizes the *affordances* of objects.
- Composition **B** and type raising **T** also show up as the defining operations of Combinatory Categorial Grammar (CCG).
- There is developmental, neuroanatomical, and neurophysiological evidence that suggests that the involvement of these operations in motor planning is a direct precursor of their involvement in natural language grammar.
- Multilayer Perceptrons, Associative Networks, and Recurrent Networks can be used to learn the building blocks of such systems.

I: The Linear Dynamic Event Calculus (LDEC)

- Basic Dynamic Logic:

  (1) $n \geq 0 \Rightarrow [\alpha](y = F(n))$

  (2) $n \geq 0 \Rightarrow \langle\alpha\rangle(y = F(n))$

- The particular dynamic logic that we are dealing with here is one that includes the following dynamic axiom (the operator ; is *sequence*, an operation related to functional composition of functions of type *situation* → *situation*):

  (3) $[\alpha][\beta]P \Rightarrow [\alpha;\beta]P$

- Composition is one of the most primitive *combinators*, or operations combining functions, which Curry and Feys (1958) call **B**, writing the above sequence $\alpha;\beta$ as $\mathbf{B}\beta\alpha$, where

  (4) $\mathbf{B}\beta\alpha \equiv \lambda s.\beta(\alpha(s))$

Dynamic Logic Actions as Accessibility

- The actions $\alpha, \beta, \ldots$ can be seen as defining the accessibility relation for a modal logic with an S4 model:

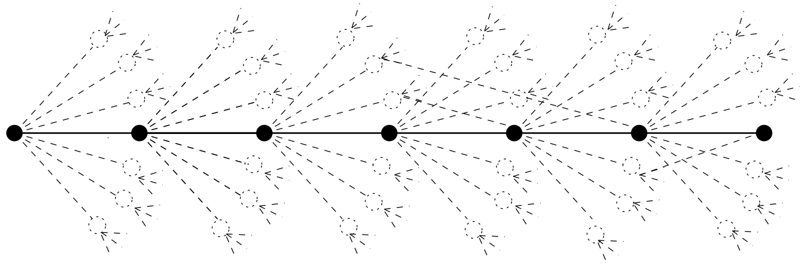


Figure 1: Kripke Model of Causal Accessibility Relation

Situation/Event Calculi and the Frame Problem

- The Situation Calculus (McCarthy and Hayes 1970) and its descendants can be seen as reified versions of Dynamic Logic.
- These calculi are heir to the "Frame Problem," which arises from the fact that humans conceptualize events in terms of very localized changes to situations.
- For example, the effects of an event of *My eating a hamburger* are confined to specific aspects of myself and the hamburger. The color of the walls, the day of the week, the leadership of the Conservative and Unionist party, and countless other aspects of the situation remain unchanged.
- This character of the knowledge representation raises the Frame Problem in two forms: the "Representational" and "Inferential" versions.

The Representational Frame Problem

- Since change is local, it is cumbersome to explicitly represent the input effect of each event on each fact by innumerable rules such as

  (5) $color(wall,x) \Rightarrow [eat(hamburger)]color(wall,x)$

- Kowalski 1979 solved the representational problem using reified Frame Axioms such as:

  (6) $p \wedge (p \neq here(hamburger)) \Rightarrow [eat(hamburger)]p$

- This keeps rules defining the positive effects of eating hamburgers simple. (Note that p is "overloaded," standing for both the fact that *p holds* and for the term p as an individual, as is standard in logic programming.)
- But if we ever need to know what the color of the walls is after a sequence of, say, five hamburger eating events, then we have to do costly theorem-proving search. This is the *Inferential* form of the Frame Problem.

The Inferential Frame Problem

- The STRIPS program (Fikes and Nilsson 1971) solved both representational and inferential problems by representing the situation as a changing *database* and representing change as sets of *preconditions* and localized *database updates*:
-

OPERATOR:	$\lambda x.eat(x)$
PRECONDITIONS:	$hamburger(x)$ $here(x)$ $hungry$
DELETIONS:	$here(x)$
ADDITIONS:	$thirsty$

- Such representations were initially derided by logicians (because of their nonmonotonicity) . . .
- . . . but then Girard (1995) came along with Linear Logic, and update was logically respectable after all!

STRIPS rules as Linear Implication

- We can represent events involving doors in this notation for a world (simplified for purposes of exposition) in which there are two places *out* and *in* separated by a door which may be *open* or *shut*, as follows:
- First we need rules to capture the fact that pushing a door changes it from open to shut and vice versa:

  (7) a. $shut(x) \multimap [push(x)]open(x)$
  b. $open(x) \multimap [push(x)]shut(x)$

  (8) a. $in \multimap [go\text{-}through(x)]out$
  b. $out \multimap [go\text{-}through(x)]in$

- These rules use *linear* logical implication $\multimap$ rather than intuitionistic implication $\Rightarrow$ in those rules that change the value of facts.

STRIPS updates as Linear Implication (Contd.)

- If the initial situation is that you are in and the door is shut:

(9) $\mathit{in} \wedge \mathit{door}(d) \wedge \mathit{shut}(d)$

—then the linear rules (7) mean that pushing the door gets you to a state where all of the following hold:

(10) a. $[\mathit{push}(d)]\mathit{open}(d)$
b. $[\mathit{push}(d)]\mathit{door}(d)$
c. $[\mathit{push}(d)]\mathit{in}$

—while the following does not hold:

(11) $[\mathit{push}(d)]\mathit{shut}(d)$

STRIPS updates as Linear Implication (Contd.)

- Using linear implication (or the equivalent rewriting logic devices or state update axioms of Thielscher (1999) and Martí-Oliet and Meseguer (1999)) for STRIPS-like rules makes frame axioms along the lines of (6) unnecessary. Instead, they are theorems of the linear logic representation.

STRIPS Preconditions as Intuitionistic Implication

- Preconditions are defined in terms of a predicate *affords*:

(12) a. $\mathit{door}(x) \wedge \mathit{open}(x)$
$\Rightarrow \mathit{affords}(\mathit{go\text{-}through}(x))$
b. $\mathit{door}(x) \Rightarrow \mathit{affords}(\mathit{push}(x))$

- We also need to define the transitive property of the possibility relation, as follows, using the definition 3 of event sequence composition:

(13) $\models \mathit{affords}(\alpha) \wedge [\alpha]\mathit{affords}(\beta) \Rightarrow \mathit{affords}(\alpha;\beta)$

STRIPS Planning in Dynamic Event Calculus

- This fragment gives us a simple planner in which starting from the world (14) in which I am *in*, and the door is *shut* and stating the goal (15) meaning “find a possible series of actions that will get me *out*,” can be made to automatically deliver a constructive proof that one such plan is (16):

(14) $\mathit{in} \wedge \mathit{door}(d) \wedge \mathit{shut}(d)$

(15) $\mathit{affords}(\alpha) \wedge [\alpha]\mathit{out}$

(16) $\alpha = \mathit{push}(d);\mathit{go\text{-}through}(d).$

- The situation that results from executing this plan in the start situation (9) is one in which the following conjunction of facts is directly represented by the database:

(17) $\mathit{out} \wedge \mathit{door}(d) \wedge \mathit{open}(d)$

II: How Animals Make Plans

- The dynamic axioms of LDEC can be viewed as a formalization of Miller et al's **TOTE units**, or of the Behaviorists notion of **operant**.
- What the logic adds is a formal way to plan with dynamic units.
- Some animals can make plans involving tools (Köhler 1925).

Figure 2: From Köhler 1925

Figure 3: From Köhler 1925

The Monkey and the Bananas

- The monkey and bananas, again simplified: grabbing something gets you to the state of having it, and if you were 6 ft higher than where you are you could grab the bananas (hack avoids axiomatizing arithmetic):

(18) a. $[grab(x)]have(x)$

b. $at((here+3)+3) \Rightarrow affords(grab(bananas))$

- If something is a box you can climb on it:

(19) $box(b) \Rightarrow affords(climb\text{-}on(b))$

- —and if you are at a place and you climb on a box you are at a place that is higher by 3ft:

(20) $at(p) \multimap [climb\text{-}on(b)]at(p+3)$

The Monkey and the Bananas, (Contd.)

- If two boxes have nothing on top of them and are not the same box you can put one on the other:

(21) $box(x) \wedge box(y) \wedge clear(x) \wedge clear(y) \wedge (x \neq y) \Rightarrow affords(puton(x,y))$

- —and if x is on something and you put it on something else then that something becomes clear and x is on that something else:

(22) a. $on(x,z) \multimap [puton(x,y)]clear(z)$
b. $clear(y) \multimap [puton(x,y)]on(x,y)$

- It is worth noting that the use of a hybrid logic in which facts like *clear* can be antecedents to both nonlinear qualification rules like (21) and linear ramification rules like (22) means that we avoid reintroducing the frame problem by having to explicitly state that $clear(x)$ in the consequent of (22b).
- Not only is this efficient—it also keeps us to Horn clause logic.

The Monkey and the Bananas, (Contd.)

- If the initial state of the world is as follows:

(23) $at(here) \wedge box(b1) \wedge box(b2) \wedge clear(b1) \wedge clear(b2)$

- —then the goal (24a) gives rise to (24b) as one possible plan

(24) a. $affords(\alpha) \wedge [\alpha]have(bananas)$
b. $\alpha = [puton(b1, here); climb\text{-}on(b1);$
$puton(b2, b1); climb\text{-}on(b2); grab(bananas)]$

- **However, we have said nothing yet about the problem of *Search* implicit in identifying such plans**.

Animals and Plans

- Such planning seems to be *reactive* to the presence of the tool and *forward-chaining*, rather than backward-chaining (working from goal to tool). That is, the animal can make a plan in the presence of the tool, but has difficulty with plans that require subgoals of finding tools.
- It implies that actions are accessed via perception of the objects that mediate them—in other words that actions are represented as the *affordances* of objects, in Gibson's (1966) terms.
- This seems a good way for an animal to plan. If there *is* a short plan using available resources, forward chaining will find it. Backward chaining is only really useful when you have evolved to the point of having devices like credit cards and mobile phones.

Formalizing Affordance in LDEC

- We can define the affordances of objects directly in terms of LDEC preconditions like (12).
- Thus the affordances of doors in our running example are pushing and going through:

(25) $affordances(door) = \left\{ \begin{array}{l} push \\ go\text{-}through \end{array} \right\}$

- The affordances of boxes are climbing on and putting on:

(26) $affordances(box) = \left\{ \begin{array}{l} climb\text{-}on \\ put\text{-}on \end{array} \right\}$

- This provides the basis for Reactive, Affordance-based, Forward-Chaining plan construction that is characteristic of primate planning.

Formalizing Affordance in LDEC (Contd.)

- The Gibsonian affordance-based door-schema can then in turn be defined as a function mapping doors into (second-order) functions from their affordances like pushing and going-through to their results:

(27) $door' = \lambda x_{door}.\lambda p_{affordances(door)}.px$

- The operation of turning an object of a given type into a function over those functions that apply to objects of that type is another primitive combinator called **T** or *type raising*, so (27) can be rewritten $door' = \lambda x_{door}.\mathbf{T}x$, where

(28) $\mathbf{T}a \equiv \lambda p.p(a)$

Type-Raising and Natural Language

- The type-raising combinator **T** is related to the notions of *Object-Orientation* and *Continuation Passing* in the theory of programming languages.
- Such a concept of doors is useful for reactive planning, and one can add more affordances to $affordances(door)$ as one's experience increases.
- However, it is a somewhat stultifying representation in human terms, One would like to have the advantages in terms of efficiency of planning that thinking of objects in terms of their affordances allows, while also being able envisage novel uses for doors—for example, using one as a table, or as a raft—when circumstances demand it.
- One of the few sources of information about the natural classifications of objects that permit limited generalization comes from linguistics.

III: Combinatory Categorial Grammar (CCG)

- Replaces PS rules by lexical categories and general combinatory rules (**Lexicalization**):

(29)

$$\xcancel{\begin{array}{lll} S & \rightarrow & NP\ \ VP \\ VP & \rightarrow & TV\ \ NP \\ TV & \rightarrow & \{proved,\ finds,\ \ldots\} \end{array}}$$

- Categories:

(30) proved := $(S\backslash NP)/NP : prove'$

Combinatory Categorial Grammar

- Combinatory Rules:

$$\frac{X/Y : f \quad Y : g}{X : f(g)} > \qquad \frac{Y : g \quad X\backslash Y : f}{X : f(g)} <$$

$$\frac{X/Y : f \quad Y/Z : g}{X/Z : \lambda z.f(g(z))} >\mathbf{B} \qquad \frac{Y\backslash Z : g \quad X\backslash Y : f}{X\backslash Z : \lambda z.f(g(z))} <\mathbf{B}$$

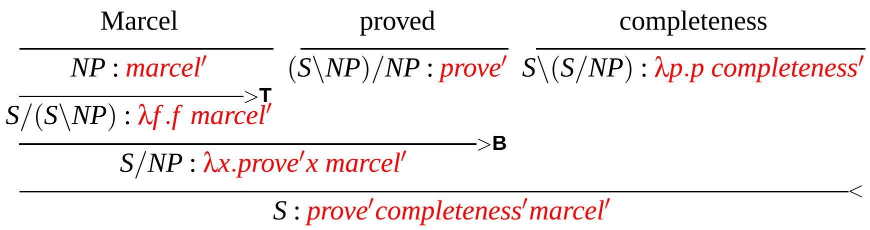
$$\frac{X/Y : f \quad Y\backslash Z : g}{X\backslash Z : \lambda z.f(g(z))} >\mathbf{B}_{\times} \qquad \frac{Y/Z : g \quad X\backslash Y : f}{X/Z : \lambda z.f(g(z))} <\mathbf{B}_{\times}$$

- All arguments are type-raised via the lexicon:

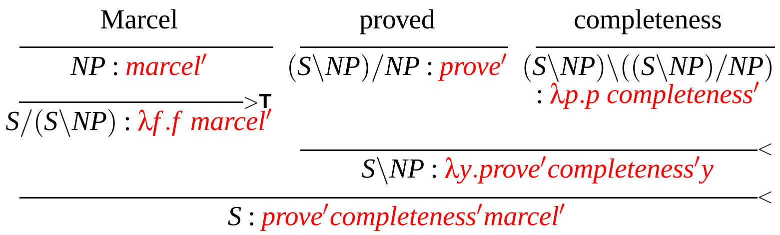
$$\frac{X : x}{\mathrm{T}/(\mathrm{T}\backslash X) : \lambda f.f(x)} >\mathbf{T} \qquad \frac{X : x}{\mathrm{T}\backslash(\mathrm{T}/X) : \lambda f.f(x)} <\mathbf{T}$$

Combinatory Derivation

(31)



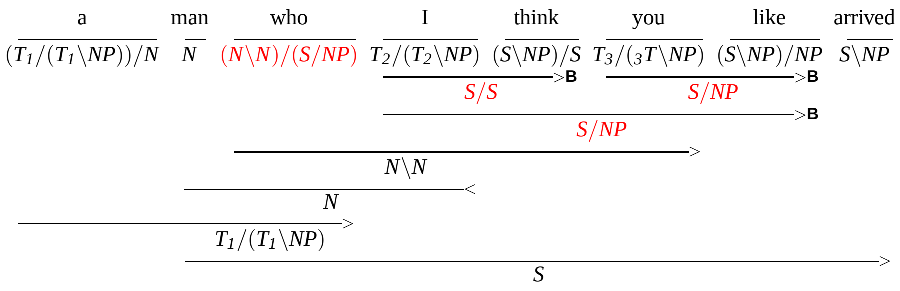
(32)



Linguistic Predictions: Unbounded "Movement"

- The combination of type-raising and composition allows derivation to project lexical function-argument relations onto "unbounded" constructions such as relative clauses and coordinate structures, without transformational rules:

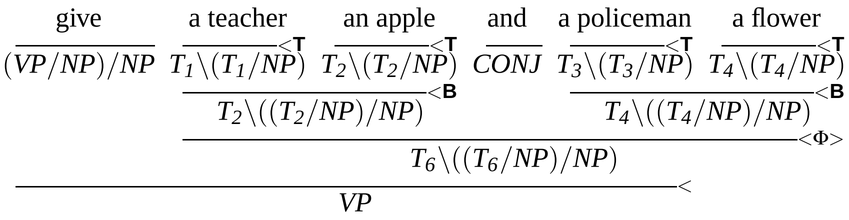
(33)



Predictions: Argument-Cluster Coordination

- The following construction is predicted on arguments of symmetry.

(34)



- A variant like the following cannot occur in an SVO language like English:

(35) *A policeman a flower and give a teacher an apple.

Syntax = Type-Raising and Composition

- CCGs combination of type-raising and composition yields a "mildly context sensitive" permuting and rebracketing calculus closely tuned to the needs of natural grammar.
- The argument cluster coordination construction (34) is an example of a universal tendency for "deletion under coordination" to respect basic word order: in all languages, if arguments are on the left of the verb then argument clusters coordinate on the left, if arguments are to the right of the verb then argument clusters coordinate to the right of the verb (Ross 1970):

(36) SVO: *SO and SVO SVO and SO
VSO: *SO and VSO VSO and SO
SOV: SO and SOV *SOV and SO

An Aside: Languages that Lexicalize Affordance

- Many North American Indian languages, such as the Athabascan group that includes Navaho, are comparatively poorly off for nouns. Many nouns for artefacts are morphological derivatives of verbs.
- For example, “towel” is *bee ’ádít’oodí*, glossed as “one wipes oneself with it”, and “towelrack” is *bee ’ádít’oodí bąąh dah náhidiiltsos*—roughly “one wipes oneself with it is repeatedly hung on it”.
- Such languages appear to lexicalize nouns as a default affordance.
- *Of course*, we should avoid crassly Whorfean inferences about Navaho-speakers abilities to reason about objects. Though productive, these lexicalizations are as conventional as our own.

Plans, Affordances, and Grammar

- The ubiquitous appearance of composition and type raising in both affordance-mediated action planning of the most elementary sort on the one hand, and universal grammar on the other, strongly suggests that the language faculty in its syntactic aspect is directly hung onto a more primitive set of prelinguistic operations originally developed for motor planning.
- The left inferior frontal (Broca’s) area that evidence from brain imaging and acquired aphasias suggests is implicated in morphosyntactic processing is immediately adjacent to areas involved in motor planning, suggesting that in evolutionary and developmental terms, the former are built upon the latter.
- The association of specific loss of *verbs* and LIF aphasias is suggestive.
- So is the fact that, when shown an object, temporal aphasic patients who cannot recover the noun (“knife”) may still be able to recover its affordances (“It is to cut with”, or related gesture, Miller et al. 1960:196).

Plan and the Onset of Language Development

- The onset of language in infants—and the entire cognitive explosion into the Piagetian operational phase—follows closely on the mastery of motor planning involving the use of tools at the final sixth stage of the Piagetian sensory-motor phase of cognitive development.
- The onset in the child of the ability to compose motor plans such as those needed for composite reaching around an obstacle anticipates the onset of productive language use. It is also argued by Deacon (1988) and Diamond (1990) to depend on the mastery of response inhibition mediated by more frontal areas that are also implicated in language disorders.
- Damage to these more frontal areas is also characteristic of long-term impaired Broca’s aphasia (Blumstein et al.)

Interim Summary

- Planning involves suppressing actions as well as seriating them.
- Planning requires a symbolic representation distinct from the motor routines that execute the plan.
- Plans are hierarchical, and partly recompiled as in Explanation-Based Learning.
- Plans are constructed by forward chaining or **B**-composition of more elementary actions.
- Plans are object-oriented, exploiting **T**-raising of objects over the actions that they afford.
- This makes the mapping of embedded motor routines to symbolic action representations the central problem of a neuroscience of planning.
- Hence it makes the central problem of language evolution and development the *verb*, and suggests attention to Broca’s area.

IV: Neural and Computational Theories

- The primate cytoarchitectonic homolog of area 44 or Broca's area in humans, F5, has been shown by single cell recording to include "Mirror Neurons" that fire not only to specific goal oriented actions such as reaching and grasping, but also (with exquisite specificity) to the sight of another animal performing the same goal-oriented action (Rizzolatti et al. 2002).
- If the animal knows that the goal is not contextually valid, or if the other animals gaze is not consistent, the mere sight of motion is not enough to fire the mirror neuron.
- Other neurons in F5 fire only to the animals own actions, and/or fire to visual presentation of the object involved (Rizzolatti et al. 2001; Miall 2003).
- This system has usually been interpreted in terms of recognition, understanding, and imitation of the actions of other animals (Gallese et al. 1996).

Origins of Symbolic Representation

- It seems likely that such understanding is founded on an even more basic capability for planning the animal's own actions, of the kind proposed above.
- In particular, it seems likely that the purely motor-sensitive neurons of F5 are closely related to rules of the LDEC type, aka TOTE units or operants.
- —and that the visual object-related neurons are related to the apparatus that associates objects with the actions that they afford (Miall 2003:2135).
- The interest of the mirror neurons themselves is then that their generalization over participant identities makes them necessarily **symbolic** representations, distinct from both efferent motor activity and afferent pure perception
- These units also look as though they would map very directly onto verbs, whether we think of these as case-frames (Rizzolatti and Arbib 1998), dependency structures (Pulvermüller 2002) or CCG lexical categories.
- In CCG, such lexical items constitute the entire language specific grammar.

A Project for a Cognitive Neurolinguistics

- This entire system is prelinguistic, rather than language-specific.
- Much of it seems to be highly localized, rather than parallel-distributed.
- However, mechanisms like Simply Recurrent Networks (SRN, Elman 1990) may well be appropriate for the process of compilation of repeated plans into compound actions and episodic memories, as opposed to novel plan construction and natural language understanding
- We need to know more about F5 in primates, specifically in relation to tool use. Are there "affordance" mirror neurons that fire both to use and appearance of tools?
- We need to understand how the planning process exploits units in F5. The limbic system seems to be implicated.

Project (Contd.)

- We need neurocomputational and machine-learning theories of how symbolic units of the kind found in F5 can be induced from sensory-motor input.
- Study of the regions adjacent to F5, (e.g. F4 which has spatially located action units Rizzolatti et al. 2002) and pathways to and from the cerebellum (Miall 2003) which executes and monitors them, are likely to be important.
- The computational character of the cortico-cerebellar-hippocampal sensory motor system is fairly well understood since Marr (1969)—see Gluck and Myers 2000.
- Perceptron-like reinforcement learning conditional on the intended goal state of LDEC-like operants seems to offer a mechanism for the neocortex and cerebellum and associative networks for the hippocampus.

Project (Contd.)

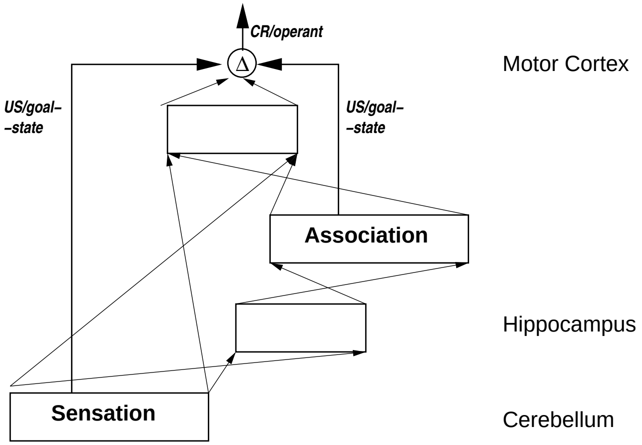


Figure 4: The basic Cerebellar-Hippocampo-Cortical dual-path circuit: (adapted from Gluck and Myers).

Project (Contd.)

- The neural pathways to and from the motor cortex remain less clear (Daskalakis et al. 2004). It is likely that several levels of plan representation mediate (Wolpert et al. 2003).
- The process of abstracting over complete action representations needed to specify the verb/affordance-like units of F5 seems to be an open problem.

Why don't Apes have Productive Syntax?

- If composition and type raising are prelinguistic primitives that we share with other animals, what more is needed to support the language faculty?
- One candidate is modal and propositional attitude concepts—that is, functions over propositional entities. (We have so far glossed over the important fact that plans compose actions of type *state* → *state*, whereas syntax composes functions of type *proposition* → *proposition*.) These induce true recursion in conceptual structures and grammar via the grounded lexicon.
- There is no evidence that apes can attain the kind of theory of other minds that is required to support such concepts. Perhaps this is *all* they lack (Premack and Premack 1983;Tomasello 1999; Steedman 2002a,b; Hauser et al. 2002).
- If so, we need to know much more about the development of propositional attitude concepts, and their relation to planning and tool use around Piagetian sensory-motor developmental stage 6.

Conclusion

- Compositionality seems to be a general property of simple sensory motor planning.
- Plan units a.k.a. STRIPS/LDEC rules seem to be learnable with standard neurocomputational models and observable with single-cell recording.
- Abstraction, plan formation and plan execution are well understood in formal terms but remain to be understood in neurocomputational terms.

Appendix: An Earlier Approach

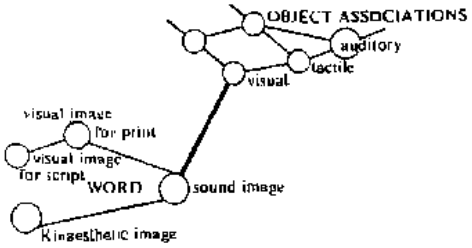


Figure 5: Freud (1891)

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