This note describes the problem of test portfolio selection/construction that arises in the context of regulators’ use of test portfolios to validate and benchmark the models firms use for calculating their regulatory capital requirements. Regulators allow the use of firms’ own models for calculation of capital requirements for several risk types and business areas – the focus of this note is on market risk in the trading book, i.e. the capital firms (mainly large banks) are required to hold against the fluctuations in value of assets held with “trading intent”. Such assets are required to be re-valued (“marked to market”) daily, it is therefore implicitly assumed that a reliable market price is available every business day, and market risk is broadly defined as the uncertainty in the future value of the product due to changes in market prices.

Firms with trading books are required to hold capital to absorb potential losses due to market moves, and under the the so-called Basel accords are allowed to use their own (“internal”) models to calculate this capital requirement, subject to regulatory approval. Regulators typically rely on firms’ internal model validation, but this is increasingly being supplemented by hypothetical portfolio exercises (where several firms are given the same portfolios to evaluate), as concerns increase about variability of capital requirements and adequacy of risk capture following the experience of the most recent financial crisis. The importance of test portfolios will increase, as regulators are becoming more interested in the cross-industry benchmarking of models and capital – this is very likely to be reflected in future versions of regulatory requirements.

The term “model” is used in two distinct ways in the present context: “valuation” models and “capital” models. Valuation models are used to calculate the value of financial products given a state of the market – note that the values of complex financial instruments are calculated as functions of the (more readily observable) values of simpler instruments, also called “risk factors”. Capital models are used to calculate capital requirements, and are based on estimating a future distribution of market values (or equivalently, changes in market values from the current value, or “P&L”) over a given time horizon, and calculating a metric (VaR, Expected Shortfall) from this distribution.

For the validation of capital models, the set of financial products to which the model applies is assumed to be known; the values of these products are in turn dependent on a set of known risk factors. Test portfolios for the model will therefore be defined either by subsets of the set of products in scope, or by subsets of the set of risk all factors, which will in turn define test portfolios of (potentially simplified) products. As a simple example, consider a product scope of interest rate swaps and FX forwards. The associated risk factors are the FX spot rate and the interest rate curve, the latter represented by a set of interest rates at different maturities. A test portfolio for this scope could consist of some combination of swaps and FX forwards, or, when analysed in terms of risk factors, might consist of portfolios of FX forwards and zero coupon bonds, the latter having exposure to just one point on the curve.

The validation of capital models can usefully be thought of as having three distinct (although obviously related) objectives:

1. Verify that risk factors for individual products are accurately captured. This is predominantly driven by the accuracy of the valuation function used for a given product in the implementation of the capital model, the main issue being that approximations (e.g. Taylor series) of valuation
functions are often used in capital models. This aspect is typically verified by comparing the actual historical P&L for the product with the corresponding P&L calculated by the capital model, i.e. product-level or risk factor level backtesting.

2. Verify that risk factor data used in the capital model is sufficiently granular, for example CDS contracts on distinct but closely related names should have distinct risk factors in the model. This aspect is typically verified through test portfolios of simple strategies (i.e. combinations of products) that give pure exposure to the risk factors being tested. For example, the CDS example could be tested by a strategy consisting of (via CDS) buying protection on one name and selling protection on the other name, with the CDS contracts having the same maturity, notional etc. The capital model should not give a capital requirement of zero for this strategy – this would indicate that the model maps both contracts to the same name.

3. Test the impact of risk factor volatility and correlation, and product combinations, on the level of capital produced by the capital model; this will be determined by the characteristics of the capital model: the type of metric (VaR, ES), the amount of data used to calibrate the model (1 year, 5 years), the framework (Historical, Monte Carlo), and weighting schemes used on risk factors or P&L distributions. There are, however, also obvious interactions with the first two test objectives. The fundamental question is whether the capital model gives adequate levels of capital during “normal” and “stressed” periods, and for all portfolios. The main (but not necessarily only) test is backtesting of capital against actual portfolio P&L, ideally compared across different model types. The crux of the test portfolio problem is that, while backtesting individual products across different models and time periods give a lot of information, the “portfolio effect” can mean that not all portfolios are equally well capitalised at all times in all models, and model validation should attempt to understand these characteristics. Different products (and asset classes) tend to become more correlated during crises, and the way correlation and volatility is modelled will have an impact on whether the model is aggressive or conservative for different portfolios at different points in time.

All three these aspects are (or should be) individually validated by firms, using a range of approaches including test portfolios. Regulatory test portfolios necessarily need to test all three, interrelated, aspects. The problem is to design a set of portfolios that does this efficiently, and is further complicated by the fact that information on valuation models and data is typically limited.

The idea is that Mechanised Reasoning could provide model validators with new ways to discover efficient test portfolios, or even help to prove a type of equivalence of different test portfolios within and across capital models.