

First in-class test

**Question 1**

Solve the following systems of linear equations by Gaussian elimination:

(a)

$$\begin{array}{rccccrcr} x_1 & + & 2x_2 & + & x_3 & = & -1 \\ 2x_1 & - & x_2 & + & x_3 & = & 1 \\ -2x_1 & + & x_2 & - & 2x_3 & = & 2 \end{array}$$

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(b)

$$\begin{array}{rccccrcr} x_1 & + & x_2 & + & x_3 & = & 0 \\ 3x_1 & + & 2x_2 & + & x_3 & = & 3 \\ x_1 & & & - & x_3 & = & 3 \end{array}$$

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**Question 2**

Consider the following five points in 3D space:

$$P_1 = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \quad Q_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \quad Q_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad Q_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

(a) Set up the parametric representation of the line  $L$  through  $P_1$  and  $P_2$ .

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(b) Set up the parametric representation of the plane  $E$  through  $Q_1$ ,  $Q_2$ , and  $Q_3$ .

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(c) Compute the normal form of  $E$ .

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(d) Compute the distance of  $P_1$  and  $P_2$  from  $E$ . Are they both on the same side of  $E$ ?

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(e) Compute the point of intersection between the line  $L$  and the plane  $E$ .

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(f) Compute the parametric representation of the reflection of the line  $L$  at the plane  $E$ .

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(g) What is the angle between the line  $L$  and the plane  $E$ ?

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**Formulas**

**Angle  $\alpha$  between vectors  $\vec{v}$  and  $\vec{w}$ :**  $\cos \alpha = \frac{\langle \vec{v}, \vec{w} \rangle}{|\vec{v}| \cdot |\vec{w}|}$

**Normal to  $\vec{v}$  and  $\vec{w}$ :**  $\vec{n} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$

**Distance of a point  $Q$  from a plane (in normal form):**  $\frac{d - \langle Q, \vec{n} \rangle}{|\vec{n}|}$

**Mirror image  $Q'$  of point  $Q$  with respect to plane (in normal form):**

$$Q' = Q + 2 \times \frac{d - \langle Q, \vec{n} \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$$