

Second in-class test

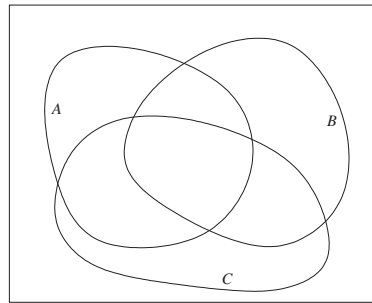
Question 1

Which of the following sets are equal to $A = \{x \in \mathbb{Z} \mid x^2 < 10\}$?

- (a) $\{0, 1, 2, 3\}$ 4%
- (b) $\{-3, -2, -1, 0, 1, 2, 3\}$ 4%
- (c) $\{0, 1, 4, 9\}$ 4%
- (d) $\{0, 1, -1, 4, -4, 9, -9\}$ 4%
- (e) $\{x \in \mathbb{Z} \mid x < 4\}$ 4%

Question 2

Copy the following Venn diagram into your notes and shade the area that corresponds to $(A \cup B) \setminus \bar{C}$. 20%



Question 3

- (a) Sketch the construction that shows that the set of all Java programs can be listed one by one, just like the natural numbers. 10%
- (b) What can you conclude from this and the fact that the set of functions from \mathbb{N} to \mathbb{N} can **not** be enumerated in this way? 10%

Question 4

Consider the following three Java method definitions. For each say whether it implements a function, and if so, whether the function is injective, surjective, or bijective. Whenever you answer is no, give a reason. 6+6+8%

- (a) `double method_a(double x) {return x+1.0;}`
- (b) `double method_b(double x) {return 2.0*x;}`
- (c) Assume that a random number generator has been created with the command `Random generator = new Random();`

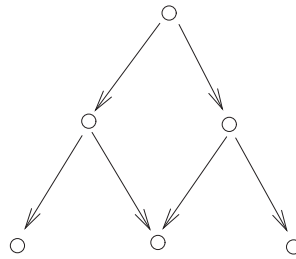
`double method_c(double x) {return x*generator.nextDouble();}`

Please turn over...

Let (V, E) be a directed graph (that is, V is a set of “nodes” and $E \subseteq V \times V$ is a set of “edges”). Let us say that two nodes are “siblings” if they have the same “ancestors” where the ancestors of a node v are those nodes from which there is an edge to v .

- (a) Copy the following graph into your answer sheet and indicate which nodes are siblings of each other.

6%



- (b) Write out a formula that states that two nodes v_1 and v_2 are siblings.

8%

- (c) Argue that “being siblings” is an equivalence relation on the set V of nodes.

6%