06-21254 *Mathematical Techniques for Computer Science*
Autumn Semester 2008

The University of Birmingham
School of Computer Science
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First in-class test

Question 1

Translate the following system of linear equations into the compact notation and solve by Gaussian elimination:

$$\begin{array}{rcrcrcr} 2x_1 & - & 3x_2 & + & x_3 & = & -1 \\ -3x_1 & + & 2x_2 & + & 2x_3 & = & 0 \\ x_1 & - & x_2 & + & x_3 & = & 1 \end{array}$$

6 points

Question 2

After running Gaussian elimination, we obtained the following echelon form:

$$\left(\begin{array}{rrrr|r} 0 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Describe the set of solutions. 4 points

Question 3

Consider the triangle in 3D with corners $P_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $P_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $P_3 = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$. Show that its three sides (given by the vectors $\overrightarrow{P_1P_2}$, $\overrightarrow{P_2P_3}$, and $\overrightarrow{P_3P_1}$) are all of the same length. 3 points

Question 4

(a) Set up the parametric representation of the plane E_1 through P_1, P_2, and P_3. 2 points

(b) A second plane E_2 is given by $X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + q \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + r \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$. Compute the line of intersection between E_1 and E_2. 4 points

Question 5

(a) Set up the normal form of the plane E_2 from the previous question. 3 points

(b) Show that E_2 cuts through the triangle of Question 3 (by showing that not all of P_1, P_2, P_3 are on the same side of E_2). 3 points

Question 6

(a) A line L in 3D is given by $X = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$. Compute its intersection point with E_2 from Question 4. 3 points

(b) Mirror L at E_2. 3 points

— turn over —

Question 7

(a) In the lecture we have looked at a method for finding the point P' on a line in 2D that is closest to a given point P. How can one solve the same problem for a line and a point in 3D? 2 points

(b) Apply your method to compute the point P' on L from Question 6a that is closest to $P = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. 3 points

total: 36 points

Formulas

Normal to $\vec{v}$ and $\vec{w}$: $\vec{n} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$

Distance of a point Q from a plane (in normal form): $\frac{d - \langle Q, \vec{n} \rangle}{|\vec{n}|}$

Mirror image Q' of point Q with respect to plane (in normal form):

$$Q' = Q + 2 \times \frac{d - \langle Q, \vec{n} \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$$