

Second in-class test

Question 1

Let A and B be subsets of a set X .

- (a) Draw a Venn diagram of the situation and shade the area that corresponds to the subset $A \setminus (B \setminus A)$.
- (b) Suggest a shorter expression that describes the same area as $A \setminus (B \setminus A)$.
- (c) Prove that your expression and $A \setminus (B \setminus A)$ are indeed always the same subset. 2+1+2 points

Question 2

- (a) Describe briefly what it means for a set to be “countable.”
- (b) Describe the argument that shows that the set of valid Java programs is countable.
- (c) It is a fact (not discussed in class) that the set of functions from \mathbb{N} to \mathbb{N} is uncountable. What is the consequence of this in light of part (b)? 2+2+1 points

Question 3

Let F be the set $\{(a, b) \in \mathbb{R} \times \mathbb{R} \mid b = 2a\}$.

- (a) Check that F satisfies the two conditions for a function from \mathbb{R} to \mathbb{R} .
- (b) Is F injective, surjective, and/or bijective?
- (c) Discuss the differences between F and the Java method

```
public float javaF (float a) { return 2*a; }
```

2+3+2 points

Question 4

Consider the set S of all Computer Science graduates.

- (a) Say that student a “is cleverer” than student b if the final mark average of a is higher than (or equal to) that of b . Show that this is *not* an order relation on S .
- (b) Say that student a is “about as clever as” student b if their mark averages differ by less than 1%. Show that this is *not* an equivalence relation.
- (c) Despite the observation in (b), the School classifies students into first, upper second, lower second, and third class. Discuss! 2+2+2 points

Question 5

Consider the following inductive definition of a set BoolExp of logical expressions:

Rule 1a “true” is a member of BoolExp .

Rule 1b “false” is a member of BoolExp .

Rule 2 If the string e belongs to BoolExp then so does the string “NOT e ”.

Rule 3 If the strings e and e' belong to BoolExp then so does the string “ e AND e' ”.

(a) Give *two different* derivations for the expression “NOT true AND false”.

(b) Consider the following inductive definition of a relation $ev \subseteq \text{BoolExp} \times \mathbb{N}$:

Rule 1a (true, 1) is a member of ev .

Rule 1b (false, 0) is a member of ev .

Rule 2 If (e, n) is a member of ev , then so is (NOT e , $1 - n$).

Rule 3 If (e, n) and (e', m) are members of ev , then so is (e AND e' , nm).

Show that ev is *not* a function.

(c) How could the problem found in (a) and (b) be remedied?

2+2+2 points