The University of Birmingham
School of Computer Science
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Solutions and mark sheet for first in-class test

Question 1

$$\left(\begin{array}{rrr|r} 1 & 3 & 2 & 1 \\ 2 & -1 & 2 & -2 \\ -1 & 4 & -2 & -1 \end{array}\right)$$ 1 point

$$\longrightarrow \left(\begin{array}{rrr|r} 1 & 3 & 2 & 1 \\ 0 & -7 & -2 & -4 \\ 0 & 7 & 0 & 0 \end{array}\right)$$ 2 points

$$\longrightarrow \left(\begin{array}{rrr|r} 1 & 3 & 2 & 1 \\ 0 & -7 & -2 & -4 \\ 0 & 0 & -2 & -4 \end{array}\right)$$ 1 point

Solutions:

$$x_3 = 2$$ 1 point

$$x_2 = (-4+2\times 2)/(-7) = 0$$ 1 point

$$x_1 = 1-2\times 2-3\times 0 = -3$$ 1 point

Question 2

(a)
$$x_3 = -2$$ 1 point

$$x_2 : \text{freely chosen}$$ 1 point

$$x_1 = \frac{1}{2}(1-(-2)+x_2) = \frac{3+x_2}{2}$$ 1 point

(b) Geometrically, the solutions form a line in 3D. 1 point

(This can be seen in two ways: the solution set has one free parameter, x_2, so it must be a one-dimensional object; also, it was obtained from intersecting two planes, each given in normal form.)

Question 3

(a) $X = P_1 + s\cdot\overrightarrow{P_1P_2} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + s\cdot\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ 2 points

(b) We need to find the value for s such that $Q = P_1 + s\cdot\overrightarrow{P_1P_2} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + s\cdot\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ becomes true. Looking at just the first coordinate, one gets the equation $1 = 2+s\times 1$ which has the solution $s=-1$. 1 point

Putting this back into the generating expression for the line L we get $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ which is indeed the point Q. 1 point

(c) We have to substitute the generating expression for L into the equation that defines the plane E:

$$(2+s)+(-2)+(1-2s) = 3$$ 1 point

from which we get the solution $s=-2$. 1 point

We use this value of s in the generating expression and get the answer: $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} - 2\cdot\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix}$ 1 point

(d) We check the length of the three sides:

2ex] $\left|\overrightarrow{T_1T_2}\right| = \left|\begin{pmatrix} -3 \\ -1 \\ -2 \end{pmatrix}\right| = \sqrt{9+1+4} = \sqrt{14}$

$$\left|\overrightarrow{T_2T_3}\right| = \left|\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}\right| = \sqrt{9+4+16} = \sqrt{29}$$

$$\left|\overrightarrow{T_3T_1}\right| = \left|\begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}\right| = \sqrt{1+4} = \sqrt{5}$$

and indeed they are all different. **1 point**

(e) $\vec{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ **1 point**

(f) $T_1' = T_1 + \frac{3-4}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{8}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 8 \\ -1 \\ 2 \end{pmatrix}$ **1 point**

$T_2' = T_2 + \frac{3-(-2)}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + \frac{5}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$ **1 point**

$T_3' = T_3 + \frac{3-7}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} - \frac{4}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} \\ -\frac{1}{3} \\ \frac{5}{3} \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 5 \\ -1 \\ 5 \end{pmatrix}$ **1 point**

Points for using the method correctly: **1 point**

(g) Comparing the length of the sides of the triangle we get:

$$\left|\overrightarrow{T_1'T_2'}\right| = \left|\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right| = \sqrt{2}$$

$$\left|\overrightarrow{T_2'T_3'}\right| = \left|\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\right| = \sqrt{2}$$

$$\left|\overrightarrow{T_3'T_1'}\right| = \left|\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\right| = \sqrt{2}$$

and indeed they are all the same. **1 point**

Total points: 25