06-21254 *Mathematical Techniques for Computer Science* — Autumn Semester 2009 — © Achim Jung

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First in-class test

Question 1

Translate the following system of linear equations into the compact notation and then solve it by Gaussian elimination:

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &= 1 \\ 2x_1 - x_2 + 2x_3 &= -2 \\ -x_1 + 4x_2 - 2x_3 &= -1 \end{aligned}$$

7 points

Question 2

After running Gaussian elimination, we obtained the following echelon form:

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

(a) Describe the set of solutions. — 3 points

(b) What geometric object do they form if every solution is viewed as a point in 3D? — 1 point

Question 3

Consider the following six points in 3D space:

$$P_1 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \quad P_2 = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad Q = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad T_1 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad T_2 = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \quad T_3 = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$$

and the plane E given in normal form by $x_1 + x_2 + x_3 = 3$

(a) Set up the parametric representation of the line L through P_1 and P_2. — 2 points

(b) Show that the point Q lies on L. — 2 points

(c) Find the point where the line L intersects the plane E. — 3 points

(d) Show that the three points T_1, T_2, and T_3 form a *scalene* triangle. — 1 point

(e) Write the normal vector of the plane E explicitly in the form $\vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$. — 1 point

(f) For each of T_1, T_2, and T_3 compute the nearest neighbour on the plane E. — 4 points

(g) Show that the three nearest neighbours form an *equilateral* triangle. — 1 point

Total points: 25

Formula

Nearest neighbour T' of point T on the plane $\langle \vec{n}, X \rangle = d$:

$$T' = T + \frac{d - \langle \vec{n}, T \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$$