The University of Birmingham
School of Computer Science
November 26, 2009

Solutions to second in-class test

Question 1

(a) $\{2,3,4,5\}$ (Double listing of elements loses one point.) 2 points

(b) $\{3,4\}$ 2 points

(c) $\{2\}$ (Leaving out the set brackets loses one point.) 2 points

(d) $\{(2,3),(2,4),(2,5),(3,3),(3,4),(3,5),(4,3),(4,4),(4,5)\}$ (Nine elements in total.) (Not expressing the elements in the format (n,m) loses one point.) 2 points

Question 2

(a) This is true, because for $n=3$ we have indeed $8=2^3$. 2 points

(b) This is false, because 8 is not a perfect square. 2 points

(c) This is false, because A contains 8 but B does not. 2 points

(d) This is false, because B contains 9 but A does not. 2 points

(e) This is false, because $A\cap B$ contains infinitely many elements, for example, 4, 16, 64, etc. (All elements of the form 2^{2n}.) 2 points

Question 3

There are several ways to answer this. One way is to list the elements of $\mathbb{Z}\times\mathbb{Z}$ in the following order: first all elements (n,m) where both n and m are between -1 and 1, then all the (new) elements where both n and m are between -2 and 2, and so on. 2 points

This gives the listing

$$\begin{aligned}\mathbb{Z}\times\mathbb{Z} \quad = \quad & \{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1),\\ & (-2,-2),(-2,-1),(-2,0),(-2,1),(-2,2),\\ & (-1,-2),(-1,2),(0,-2),(0,2),(1,-2),(1,2),\\ & (2,-2),(2,-1),(2,0),(2,1),(2,2),\dots\}\end{aligned}$$

2 points

Question 4

(a) (i) The relation is not reflexive because it does not contain (c,c), for example. 1 point

(ii) The relation is not symmetric because it does not contain (a,c) although it contains (c,a). 1 point

(iii) The relation is not transitive because it does not contain (f,b) although it contains (f,d) and (d,b). 1 point

(b) $$\begin{aligned}\text{equiv-closure}(R) \quad = \quad & \{(a,a),(c,c),(e,e),(a,c),(c,a),(a,e),(e,a),(c,e),(e,c),\\ & (b,b),(d,d),(f,f),(b,d),(d,b),(d,f),(f,d),(b,f),(f,b),\\ & (g,g)\}\end{aligned}$$ 2 points

(c) There are three equivalence classes: 3 points

$$\begin{aligned}[a]_\approx &= \{a,c,e\}\\ [b]_\approx &= \{b,d,f\}\\ [g]_\approx &= \{g\}\end{aligned}$$

Question 5

(a) Definedness: This is satisfied; for each of a, b, c, d, e, f, and g there is a pair in the relation with that value in first place. 1 point

Singlevaluedness: This is also satisfied because there is exactly one such pair in R. 1 point

(b) It is not injective, because both a and c are mapped to a. 2 points

It is not surjective, because no element is mapped to c. 2 points

It is therefore not bijective. 2 points

Question 6

(a) It is reflexive because $f(x) = f(x)$ (because functions are singlevalued) and so $x \approx x$. 1 point

It is symmetric because if $x \approx x'$ then by definition this is because $f(x) = f(x')$. But then one also has $f(x') = f(x)$ and therefore $x' \approx x$. 1 point

It is transitive because $x \approx x' \approx x''$ means that $f(x) = f(x') = f(x'')$ must be true, and therefore also $f(x) = f(x'')$, and so $x \approx x''$. 1 point

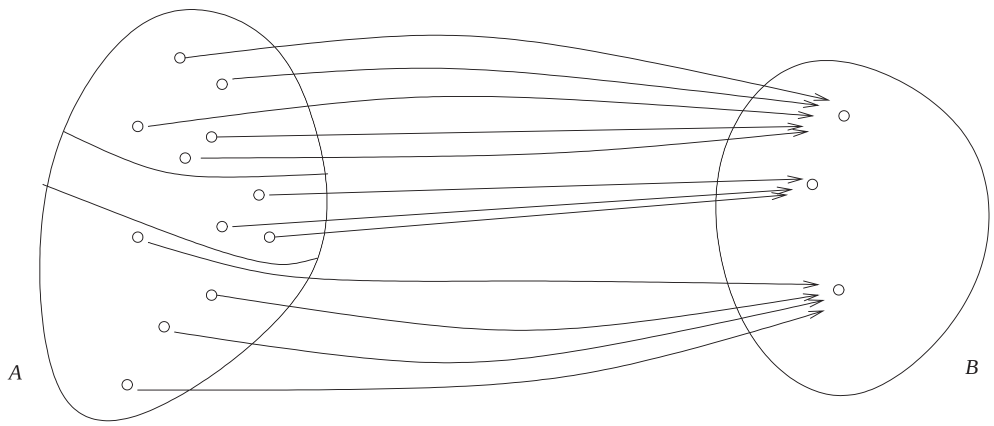
(b) By surjectivity, for every $b \in B$ we have an element $x_b \in A$ with $f(x_b) = b$. 1 point

The equivalence class $[x_b]_\approx$ consists of exactly those elements that are mapped to b by f (singlevaluedness of f. 1 point

Every element of A belongs to some equivalence class (definedness of f). 1 point

Together this shows that there is a one-to-one correspondence between the equivalence classes of $\approx$ and the elements of B, hence they have the same number of elements.

(c)



2 points

(d) (i) $f(0,0,0,0,0,0,0,0) = 0$ and $f(0,0,0,0,0,0,0,1) = 1$, so both elements of Bit are reached by the function. 1 point

(ii) The bytes that are mapped to 0 are those that have an even number of 1's, those that are mapped to 1 are those that have an odd number of 1's. 2 points

(iii) The function is known as "parity check" or "parity bit". 1 point

Total points: 50