The University of Birmingham
School of Computer Science
November 26, 2009

Second in-class test

Question 1

Let $A = \{2,3,4\}$ and $B = \{3,4,5\}$. List the elements of the sets

(a) $A \cup B$ — 2 points

(b) $A \cap B$ — 2 points

(c) $A \setminus B$ — 2 points

(d) $A \times B$ — 2 points

Question 2

Let $A = \{x \in \mathbb{N} \mid \exists n \in \mathbb{N}.\ x = 2^n\}$ and $B = \{x \in \mathbb{N} \mid \exists n \in \mathbb{N}.\ x = n^2\}$. Which of the following statements are true, which are false? Give (short!) justifications for your answers

(a) $8 \in A$ — 2 points

(b) $8 \in B$ — 2 points

(c) $A \subseteq B$ — 2 points

(d) $B \subseteq A$ — 2 points

(e) $A \cap B = \emptyset$ — 2 points

Question 3

Argue that $\mathbb{Z} \times \mathbb{Z}$ is a countable set. — 4 points

Question 4

Let $A = \{a,b,c,d,e,f,g\}$ and $R = \{(a,a),(b,b),(c,a),(d,b),(e,a),(f,d),(g,g)\}$.

(a) Which of the three conditions for an equivalence relation are satisfied by R? (Give brief justifications.) — 3 points

(b) Compute the smallest equivalence relation $\approx$ containing R. — 2 points

(c) Describe the equivalence classes derived from $\approx$. — 3 points

— turn over —

Question 5

Consider again the set A and the relation $R \subseteq A \times A$ from the previous question.

(a) Argue that it is a function from A to A. 2 points

(b) Is the function injective? surjective? bijective? (Give brief justifications.) 6 points

Question 6

Let A and B be finite sets, and f a function from A to B.

(a) Define a relation $\approx$ on A by setting

$$x \approx x' \quad \text{if} \quad f(x) = f(x')$$

and argue that it is an equivalence relation. 3 points

(b) Assume now that f is surjective and argue that in this case the number of equivalence classes induced by $\approx$ on A is the same as the number of elements of B. 3 points

(c) Draw a small diagram illustrating this fact. 2 points

(d) Concretely, let Bit be the set $\{0,1\}$ and consider $A = \mathsf{Bit}^8$ ("bit vectors of length eight" or "bytes") and $B = \mathsf{Bit}$. The function f from A to B is given by

$$f(b_1, b_2, \ldots, b_8) = b_1 \text{ xor } b_2 \text{ xor } \ldots \text{ xor } b_8$$

(i) Show that f is surjective. 1 point

(ii) According to parts (a) and (b), this induces an equivalence relation on A with two equivalence classes. Give alternative descriptions of them. 2 points

(iii) What is the usual name given to f? 1 point

Total points: 50