The University of Birmingham
School of Computer Science
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First in-class test

Question 1

Translate the following system of linear equations into the compact notation and then solve it by Gaussian elimination:

$$\begin{array}{rcr} x_2 + x_3 & = & 1 \\ 3x_1 - 2x_3 & = & 3 \\ x_1 + x_2 & = & -1 \end{array}$$

7 points

Question 2

After running Gaussian elimination, we obtained the following echelon form:

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

(a) Describe the set of solutions. 3 points

(b) What geometric object do they form if every solution is viewed as a point in 3D? 1 point

Question 3

Consider the following two points in 3D space:

$$P_1 = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \quad P_2 = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$$

and the plane E given in normal form by $x_1 - 2x_2 + x_3 = 1$

(a) Write out the normal to E explicitly in the form $\vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$. 1 point

(b) Show that P_1 and P_2 are equally far away from E but lie on opposite sides. 3 points

(c) Set up the parametric representation of the line L through P_1 and P_2. 2 points

(d) Find the point where the line L intersects the plane E. 3 points

(e) Mirror the line L at E. 3 points

Total points: 23

Formulas

Distance of a point P from the plane $\langle \vec{n}, X \rangle = d$: $\dfrac{d - \langle \vec{n}, P \rangle}{|\vec{n}|}$

Mirror image P' of point P w.r.t the plane $\langle \vec{n}, X \rangle = d$: $P' = P + 2\dfrac{d - \langle \vec{n}, P \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$