

Second in-class test

Question 1

Let $A = \{1, 2, 3\}$ and $B = \{-1, -2, -3\}$. List the elements of the sets

- (a) $A \cup B$ 2 points
- (b) $A \cap B$ 2 points
- (c) $A \setminus B$ 2 points
- (d) $A \times B$ 2 points

Question 2

Let $A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid -3 \leq x + y \leq 3\}$ and $B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x^2 + y^2 \leq 5\}$. Which of the following statements are true, which are false? Give (short) justifications for your answers.

- (a) $(2, -1) \in A$ 1 point
- (b) $(2, -1) \in B$ 1 point
- (c) $A \subseteq B$ 2 points
- (d) $B \subseteq A$ 2 points
- (e) $A \cap B = \emptyset$ 2 points

Question 3

- (a) Argue that the set of lists, whose entries are either 0 or 1 and whose length is finite, is countable. 3 points
- (b) Argue that the set of infinite lists, whose entries are either 0 or 1, is uncountable. 3 points

Question 4

Let $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, a), (b, b)\}$.

- (a) Which of the three conditions for an equivalence relation are satisfied by R ? (Give brief justifications.) 3 points
- (b) Compute the smallest equivalence relation \approx containing R . 2 points
- (c) Describe the equivalence classes derived from \approx . 3 points

Question 5

Consider the following three Java method definitions. For each say whether it implements a function, and if so, whether the function is injective and/or surjective. Whenever your answer is no, give a reason. If you are unsure about Java's definition of the datatypes `int` and `String` then state your assumptions.

(a) `String method_a(String x) {return x + "aa";}`

4 points

(b) `int method_b(String x) {return x.length();}`

4 points

(c) `String method_c(String x) {return Time() + x;}`

4 points

(Assume that `Time()` is a method that returns the current actual time as a string.)

Question 6

Let $M_{(2,2)}$ be the set of 2×2 matrices. For A and A' two 2×2 matrices, say that "A is similar to B" (and write $A \approx B$ in that case) if there is an invertible 2×2 matrix B such that $A' = BAB^{-1}$.

(a) Show that \approx is an equivalence relation on $M_{(2,2)}$.

6 points

(b) Find two matrices that are *not* similar to each other.

2 points

Total points: 50