

Exercise Sheet 1

Exercise 1.1

For each of the following systems of linear equations, transcribe into the compact notation and solve by Gaussian elimination:

(a)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_1 - x_2 + 4x_3 &= 0 \\ 2x_1 + x_2 + 5x_3 &= 3 \end{aligned}$$
 2 points

(b)
$$\begin{aligned} 2x_1 + 2x_2 + x_3 &= -1 \\ -x_1 - x_2 + 2x_3 &= 3 \\ x_1 + x_2 + 3x_3 &= 3 \end{aligned}$$
 2 points

(c)
$$\begin{aligned} -4x_1 - x_2 - x_3 &= -3 \\ 2x_1 - 2x_2 - x_3 &= 4 \\ 2x_1 + 3x_2 + 2x_3 &= -1 \end{aligned}$$
 2 points

Exercise 1.2

For each of the following systems, say whether it is in echelon form and if so, describe the set of solutions:

(i) $\left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$ (ii) $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$ (iii) $\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{array} \right)$

4 points

Exercise 1.3

A system of linear equations is called *homogeneous* if the numbers on the right hand side are all equal to zero. An example of a homogeneous system is the following (in the compact notation):

$$\left(\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & 1 & -2 & 0 \\ -2 & 4 & 0 & 0 \end{array} \right)$$

You don't need to solve this; argue generally that for a homogeneous system the "Special Base Case 1" will not occur.

2 points

Total points: 12

Stretcher Exercise 1

(You can earn two *bonus points* by answering this question. Send your solution via email directly to O.K.Klinke@cs.bham.ac.uk.)

Assume a, b, c, d are four numbers, *all different from each other*, and we are given the system of linear equations

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= r_1 \\ ax_1 + bx_2 + cx_3 + dx_4 &= r_2 \\ a^2x_1 + b^2x_2 + c^2x_3 + d^2x_4 &= r_3 \\ a^3x_1 + b^3x_2 + c^3x_3 + d^3x_4 &= r_4 \end{aligned}$$

Argue that there is always exactly one solution (independently of the value of r_1, r_2, r_3, r_4), in other words, none of the special cases will occur when running Gaussian elimination on this.