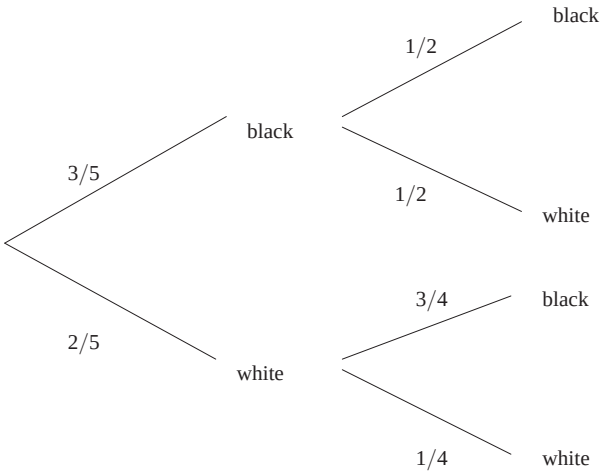
Solutions to Exercise Sheet 10

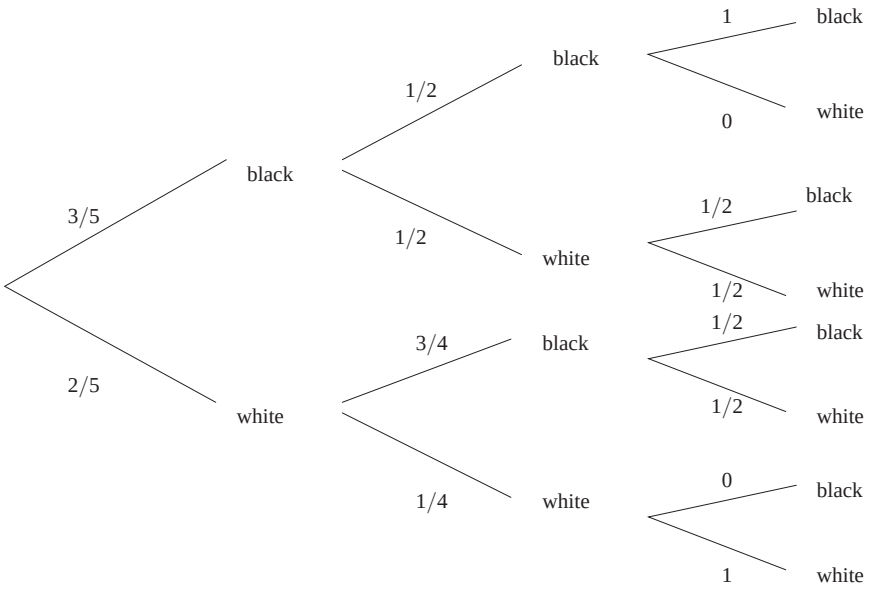
Exercise 10.1

(a)



(b) We have to compute and add up the probabilities for the outcomes "(white, black)" and "(black, white)": $\frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{1}{2} = \frac{3}{5}$

(c)



Again we compute the probabilities of all paths that end up in "white" and add them together: $\frac{3}{5} \times \frac{1}{2} \times 0 + \frac{3}{5} \times \frac{1}{2} \times \frac{1}{2} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{2} + \frac{2}{5} \times \frac{1}{4} \times 1 = \frac{2}{5}$.

Exercise 10.2

It is much easier to think about the complement event, the one where we receive no face card at all and to subtract this number from 1. We get:

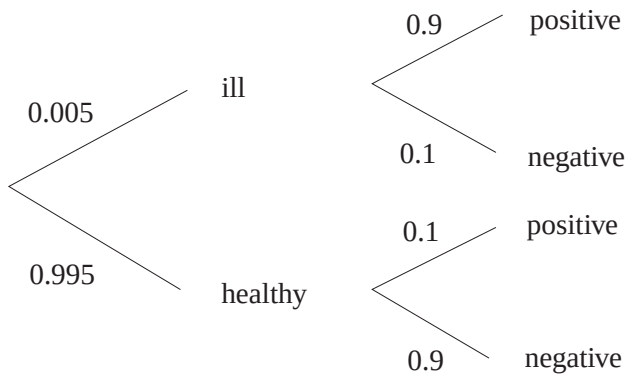
$$1 - \frac{40}{52} \times \frac{39}{51} \times \frac{38}{50} \times \frac{37}{49} \times \frac{36}{48} \approx 0.747$$

Exercise 10.3

We use the formula for two events thrice:

$$\begin{aligned} p(A\cup B\cup C) &= p(A\cup B)+p(C)-p((A\cup B)\cap C) \\ &= p(A\cup B)+p(C)-p((A\cap C)\cup(B\cap C)) \\ &= p(A)+p(B)-p(A\cap B)+p(C)-(p(A\cap C)+p(B\cap C)-p(A\cap B\cap C)) \\ &= p(A)+p(B)+p(C)-p(A\cap B)-p(A\cap C)-p(B\cap C)+p(A\cap B\cap C) \end{aligned}$$

Exercise 10.4



The probability for someone to test positive is

$$0.005\times 0.9+0.995\times 0.1=0.0045+0.0995=0.104$$

that is, about 10%. Of these 10.4% only 0.45% came from people actually ill, which is a proportion of

$$\frac{0.45}{10.4}\approx 0.04.3$$

That is, of those people who test positively, only 4.3% are actually ill, whereas 95.7% are in fact healthy!

You can do the same computation for a test that is 99% accurate and then you will find that about 1.5% of the population will test positively of which about a third are actually ill. So you still have the problem that two thirds of those that test positively are not ill at all.

In the end, it will depend on the seriousness of the illness whether it is ethical to recommend the test to everyone independent of any other symptoms of the illness.