Exercise Sheet 10

Exercise 10.1

Consider an urn with 3 black and 2 white balls inside. Two balls are drawn at random (without returning the first one).

(a) Draw the probability tree for this experiment. 2 points

(b) Compute the probability that we end up with a white and a black ball. 2 points

(c) Next, the experiment is continued: The two balls that have been drawn are put into a new empty urn and then from this a ball is drawn at random. Compute the overall probability that we end up with a white ball. 2 points

Exercise 10.2

Consider a standard 52-card deck. 12 of these are "face cards" (King, Queen, Jack). You are dealt 5 cards. What is the probability of receiving *at least one* face card? 2 points

Exercise 10.3

Develop a formula for $p(A \cup B \cup C)$ that allows one to compute this probability in case the probabilities $p(A)$, $p(B)$, $p(C)$, $p(A \cap B)$, $p(A \cap C)$, $p(B \cap C)$, and $p(A \cap B \cap C)$ are all known.
(Note: Use the formula $p(A \cup B) = p(A) + p(B) - p(A \cap B)$ that we derived in class. 2 points

Exercise 10.4

Consider an early diagnosis test for a certain illness which afflicts about 0.5% of the population. In 90% of all cases the test gives the correct diagnosis.

(a) Develop a probability tree for this situation. 2 points

(b) What is the overall probability that some random person test positive for the illness? 2 points

(c) Which proportion of the people who test positive are actually ill, which proportion consists of "false positives"? 2 points

Total points: 16

Stretcher Exercise 10

(You can earn two *bonus points* by answering this question. Send your solution via email directly to `O.K.Klinke@cs.bham.ac.uk`.)

Alice and Bob agree to meet at Starbucks some time between 4 and 4:30 later this afternoon. If Alice gets there and Bob isn't around she'll wait for 5 minutes and then leave if he hasn't arrived by then. If Bob gets there and doesn't find Alice, he'll wait 7 minutes for her and then leave if she hasn't arrived by then. Neither will wait until after 4:30. What is the probability that they will meet up, assuming that their arrival time is uniformly distributed between 4 and 4:30, and Alice's and Bob's arrival are independent of each other.

You can earn the bonus points by doing one of the following: Either

- derive the exact probability for Alice and Bob meeting mathematically (actually, geometrically), or
- write a Java program that simulates 10,000 such afternoons and gives the average of the two meeting each other.