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Solutions to Exercise Sheet 11

Exercise 11.1

(a) The sample space has nine elements; the first ball can have any of the three colours as can the second one: $S = \{(\text{red},\text{red}),(\text{red},\text{green}),(\text{red},\text{blue}),(\text{green},\text{red}),(\text{green},\text{green}),(\text{green},\text{blue}),(\text{blue},\text{red}),(\text{blue},\text{green}),(\text{blue},\text{blue})\}$.

(b) The possible outcomes are 0, 1, and 2.

(c) The probability of having no green ball: $p(X=0) = \frac{7}{10}\times\frac{6}{9} = \frac{42}{90} \approx 0.467$

The probability of having one green ball: $p(X=1) = \frac{3}{10}\times\frac{7}{9}+\frac{7}{10}\times\frac{3}{9} = \frac{42}{90} \approx 0.467$

The probability of having two green balls: $p(X=2) = \frac{3}{10}\times\frac{2}{9} = \frac{6}{90} \approx 0.067$

(d) $\mathrm{E}[X] = 0\times\frac{42}{90}+1\times\frac{42}{90}+2\times\frac{6}{90} = \frac{54}{90} = 0.6$

$\mathrm{Var}[X] = (0-0.6)^2\times\frac{42}{90}+(1-0.6)^2\times\frac{42}{90}+(2-0.6)^2\times\frac{6}{90} = 0.36\times\frac{42}{90}+0.16\times\frac{42}{90}+1.96\times\frac{6}{90} = \frac{0.52\times42+1.96\times6}{90} = \frac{33.6}{90} \approx 0.373$

$\mathrm{StdDev}[X] = \sqrt{\mathrm{Var}[X]} \approx \sqrt{0.373} \approx 0.611.$

Exercise 11.2

According to the formula for the binomial variable, we get $p(X\geq7) = \binom{10}{7}\times(\frac{1}{2})^7\times(\frac{1}{2})^3+\binom{10}{8}\times(\frac{1}{2})^8\times(\frac{1}{2})^2+\binom{10}{9}\times(\frac{1}{2})^9\times(\frac{1}{2})^1+\binom{10}{10}\times(\frac{1}{2})^{10}\times(\frac{1}{2})^0 = 120\times\frac{1}{1024}+45\times\frac{1}{1024}+10\times\frac{1}{1024}+1\times\frac{1}{1024} = \frac{176}{1024} \approx 0.172$

Exercise 11.3

The rate of arrival is $\lambda = 2$ and the corresponding probability function for the Poisson variable is $p(X=i) = \frac{2^i}{i!}\times e^{-2}$.

(a) Evaluating the formula for $i=0$ gives $p(X=0) = \frac{2^0}{0!}\times e^{-2} = \frac{1}{1}\times e^{-2} = e^{-2} \approx 0.135$

(b) Evaluating the formula for $i=4$ gives $p(X=4) = \frac{2^4}{4!}\times e^{-2} = \frac{16}{24}\times e^{-2} = \frac{2}{3}\times e^{-2} \approx 0.09$

(c) We also compute the probability for receiving 1, 2, or 3 calls:

$p(X=1) = \frac{2^1}{1!}\times e^{-2} = \frac{2}{1}\times e^{-2} = 2\times e^{-2} \approx 0.27$

$p(X=2) = \frac{2^2}{2!}\times e^{-2} = \frac{4}{2}\times e^{-2} = 2\times e^{-2} \approx 0.27$

$p(X=3) = \frac{2^3}{3!}\times e^{-2} = \frac{8}{6}\times e^{-2} = \frac{4}{3}\times e^{-2} \approx 0.18$

The probability of receiving more than four calls is 1 minus the probabilities for 0, 1, 2, 3, or 4 calls: $p(X\geq4) \approx 1-(0.135+0.27+0.27+0.18+0.09) = 1-0.945 = 0.055$. In other words, there is an 5.5% chance to receive more than 4 calls.