The University of Birmingham
School of Computer Science
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Exercise Sheet 2

Exercise 2.1

Consider the following points in 3D space

$$P_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad P_3 = \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}$$

and show that they form a *scalene* triangle (i.e., a triangle in which no two sides have the same length). 3 points

Exercise 2.2

The laws of vector algebra in Item 19 do not mention subtraction of vectors. The reason is that we can define subtraction with the help of scalar multiplication:

$$\vec{v} - \vec{w} \stackrel{\text{def}}{=\!=} \vec{v} + (-1) \cdot \vec{w}$$

(a) Draw a diagram of $\vec{v} - \vec{w}$ in the style of Box 23. 1 point

(b) Using only the laws listed in Item 19, prove the following rules about vector subtraction:

(i) $\vec{v} - \vec{v} = \vec{0}$ 1 point

(ii) $s \cdot (\vec{v} - \vec{w}) = s \cdot \vec{v} - s \cdot \vec{w}$ 2 points

Exercise 2.3

(a) Show that the following two lines in 3D intersect and compute the point of intersection.

$$X = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

2 points

(b) On the other hand, the following two lines in 3D do *not* intersect. Prove this by *attempting* to compute the point of intersection.

$$X = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \qquad Y = \begin{pmatrix} 2 \\ -4 \\ -3 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

2 points

Exercise 2.4

(a) The points

$$P_1 = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} \quad P_2 = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \quad P_3 = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

define a plane E in 3D. Give its parametric representation. 2 points

(b) Let l be the line given by

$$X = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + r \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

Compute its point of intersection with the plane E. 2 points

Total points: 15

— over —

Stretcher Exercise 2

(You can earn two *bonus points* by answering this question. Send your solution via email directly to `O.K.Klinke@cs.bham.ac.uk`.)

The following two expressions define two lines in 3D:

$$X = \begin{pmatrix} -1 \\ -4 \\ 4 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Find the shortest distance between them.