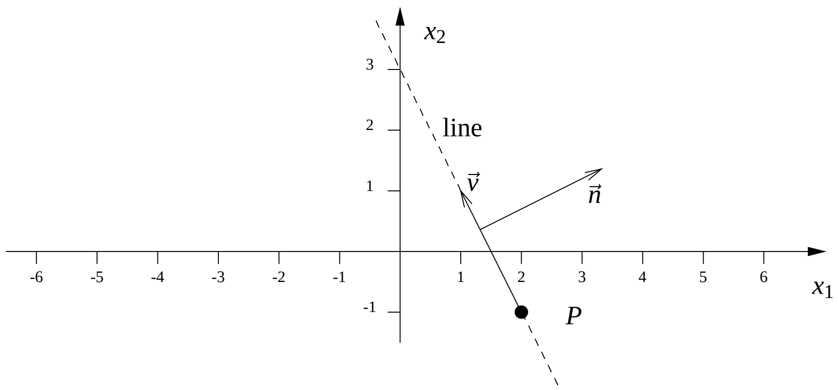
06-21254 *Mathematical Techniques for Computer Science*
Autumn Semester 2011

The University of Birmingham
School of Computer Science

Solutions to Exercise Sheet 3

Exercise 3.1

Change the direction vector $\vec{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ to the normal vector: $\vec{n} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Set up the normal form for the line as $\langle \vec{n}, X \rangle = \langle \vec{n}, P \rangle$ with $P = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ gives us $2x_1 + x_2 = 4 - 1 = 3$. Picture:



Exercise 3.2

The parametric presentation derived from three points is computed as $X = P + s \cdot \overrightarrow{PQ} + t \cdot \overrightarrow{PR}$, so we get $X = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$. Too many minus signs confuse me, so I take the negative of the direction vectors instead; this doesn't change the plane one bit: $X = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + s \cdot \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$. The coefficients of the normal form $ax_1 + bx_2 + cx_3 = d$ compute as

$$\begin{array}{lcllcr} a & = & 4-5 & & = & -1 \\ b & = & -(-4-10) & & = & 14 \\ c & = & -2-4 & & = & -6 \\ d & = & (-1) \times 1 + 14 \times 2 - 6 \times 2 & & = & 15 \end{array}$$

and the normal form is obtained as $-x_1 + 14x_2 - 6x_3 = 15$.

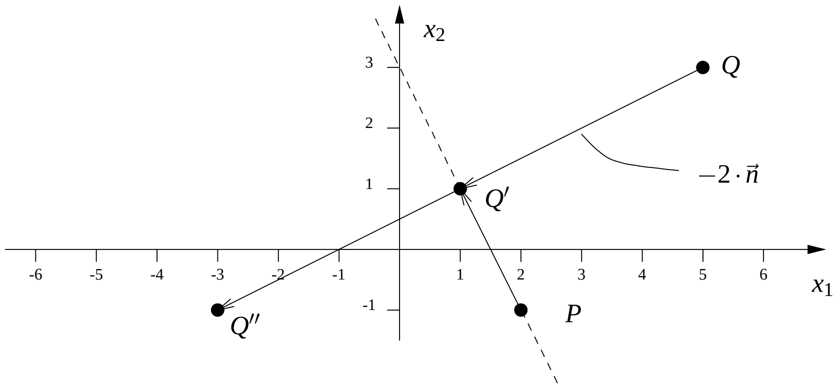
Exercise 3.3

(a) According to the formula on handout 6, the distance is computed as $\frac{d - \langle \vec{n}, Q \rangle}{|\vec{n}|} = \frac{3-(10+3)}{\sqrt{2^2+(-1)^2}} = \frac{-10}{\sqrt{5}}$.

(b) For this we compute $Q + \frac{d - \langle \vec{n}, Q \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \frac{10}{5} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(c) For this we compute $Q + 2 \times \frac{d - \langle \vec{n}, Q \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \frac{20}{5} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$.

(d) We use the inner product to check that $\vec{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ is orthogonal to $\overrightarrow{Q''Q} = \begin{pmatrix} 5-(-3) \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$:
Indeed, $\left\langle \overrightarrow{Q''Q}, \vec{v} \right\rangle = 8 \times (-1) + 4 \times 2 = 0$



Exercise 3.4

Two points on the line are

$$\text{(choosing } s=0) \quad A=\begin{pmatrix}2\\2\\-1\end{pmatrix} \qquad \text{and} \qquad \text{(choosing } s=1) \quad B=\begin{pmatrix}3\\4\\1\end{pmatrix}$$

Reflecting these yields (using $d=1$ and $\vec{n}=\begin{pmatrix}1\\-3\\0\end{pmatrix}$)

$$A' = A + 2\times\frac{d-\langle\vec{n},A\rangle}{\langle\vec{n},\vec{n}\rangle}\cdot\vec{n} = \begin{pmatrix}2\\2\\-1\end{pmatrix} + 2\times\frac{1-(2-6)}{10}\cdot\begin{pmatrix}1\\-3\\0\end{pmatrix} = \begin{pmatrix}3\\-1\\-1\end{pmatrix}$$

$$\text{and} \qquad B' = B + 2\times\frac{d-\langle\vec{n},B\rangle}{\langle\vec{n},\vec{n}\rangle}\cdot\vec{n} = \begin{pmatrix}3\\4\\1\end{pmatrix} + 2\times\frac{1-(3-12)}{10}\cdot\begin{pmatrix}1\\-3\\0\end{pmatrix} = \begin{pmatrix}5\\-2\\1\end{pmatrix}$$

Connecting A' and B' gives the parametric representation

$$X = \begin{pmatrix}3\\-1\\-1\end{pmatrix} + s\cdot\begin{pmatrix}2\\-1\\2\end{pmatrix}$$

for the reflected line.