

Exercise Sheet 3

Exercise 3.1

Compute the normal form of the line $X = P + s \cdot \vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

Draw a picture in a coordinate system that shows P , \vec{v} , and the normal \vec{n} .

3 points

Exercise 3.2

Given the following three points

$$P = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \quad Q = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} \quad R = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

find the parametric representation of the plane determined by them, and convert this into the normal form.

2 points

Exercise 3.3

Consider the line from Question 3.1.

- Compute the distance of $Q = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ from it.
- Find the point Q' on the line that is nearest to Q .
- Reflect Q at the line to obtain a point Q'' .
- Check that the line from Q'' to Q is orthogonal to the given line.
- Use the diagram from Question 3.1 to check your answers.

5 points

Exercise 3.4

Reflect the line $X = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ at the plane $x_1 - 3x_2 = 1$.

4 points

Total points: 14

Stretcher Exercise 3

(You can earn two *bonus points* by answering this question. Send your solution via email directly to O.K.Klinke@cs.bham.ac.uk.)

The corner of a “billiard table” is given by the area above the x -axis and to the left of the line $X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Ball A is located at $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, ball B at $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$. In which direction does the player have to push A so that it is reflected at both sides and hits B head-on? (Draw a picture of the situation first.)