

Exercise Sheet 4

Exercise 4.1

Assuming both A and B are $n \times n$ matrices, simplify the expression $(A + B)^3$.

2 points

Exercise 4.2

Consider the two matrices

$$A = \begin{pmatrix} -1 & 2 & -3 \\ 0 & 1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & -1 \end{pmatrix}$$

Compute AB and BA .

2 points

Exercise 4.3

Let A be the 2×2 matrix $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$.

(a) Compute A^2 , A^3 , A^4 , and A^5 .

2 points

(b) How would you describe the entries of A^n ?

1 point

Exercise 4.4

Consider the following four points in the $x - y$ -plane:

$$P_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad P_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad P_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad P_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Let A be the 2×2 matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(a) Draw a 2D coordinate system and mark the rectangle whose corners are given by the four points.

1 point

(b) Compute the matrix products AP_1 , AP_2 , AP_3 and AP_4 and interpret the results as coordinates of points P'_1 , P'_2 , P'_3 and P'_4 .

Draw the polygon determined by them into a new coordinate system.

2 points

(c) Repeat two more times.

2 points

(d) Describe the action of the matrix on points in the plane geometrically.

1 point

Total points: 13

Stretcher Exercise 4

(You can earn two *bonus points* by answering this question. Send your solution via email directly to `O.K.Klinke@cs.bham.ac.uk`.)

Write a computer program to compute how the operating system example on Handout 7, Box 48, evolves over time. Note how the entries of the stochastic matrix stabilise. Guess their limit values (they are simple fractions) and check that the corresponding matrix remains unchanged from one time slice to the next.