

Exercise Sheet 5

Exercise 5.1

Invert the matrix $A = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 0 & -1 \\ -2 & -2 & 1 \end{pmatrix}$ and check your result by computing both AA^{-1} and $A^{-1}A$.

4 points

Exercise 5.2

Solve the matrix equation $AX = B$ where X is the unknown 3×4 matrix, A is the same as in the previous question and

$$B = \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 3 & -1 & 4 \\ -2 & 0 & 1 & 3 \end{pmatrix}$$

(Check first that this makes sense in terms of the dimensions of the matrices involved.)

3 points

Exercise 5.3

[From the May 2009 exam] Let A and B be the matrices

$$A = \frac{1}{7} \cdot \begin{pmatrix} 3 & -1 & -2 \\ 4 & ? & 2 \\ -2 & 3 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ -2 & 1 & -1 \end{pmatrix}$$

Fill in the missing value in A so that it becomes the inverse of B . Check the result by computing AB .

2 points

Exercise 5.4

(This is a continuation of the example from the lecture.) Show that if a fourth type of LED is added to every pixel, which excites the eye cone cells according to the vector $\begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$ then the excitation pattern $\begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}$ (which required “negative illumination” of the blue LED in the existing system of three LEDs) can be realised.

2 points

Total points: 11

Stretcher Exercise 5

(You can earn two *bonus points* by answering this question. Send your solution via email directly to O.K.Klinke@cs.bham.ac.uk.)

- (a) Let A be a $n \times n$ matrix and p a $n \times 1$ column vector. Assume that not all entries in p are zero, and that $Ap = 0$. Show that A cannot be invertible.
- (b) Use this to show that the following matrix is never invertible, no matter how a, b, c, d, e, f, g and h are chosen:

$$\begin{pmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{pmatrix}$$