Solutions to Exercise Sheet 6

Exercise 6.1

(a) $B \cup C = \{-4,-2,0,1,2,3,4\}$ (b) $B \cap C = \{0,2,4\}$ (c) $\overline{B} \cap D = \{-1,1\}$
(d) $B \cap \overline{D} = \{-4,-2,2,4\}$ (e) $B \setminus D = \{-4,-2,2,4\}$ (f) $D \setminus B = \{-1,1\}$

(Note that (c) and (f) must give the same answer; similarly (d) and (e).)

Exercise 6.2

1. Yes, because for $n = 4$ we have $9 = 2 \times 4 + 1$.
2. Yes, because for $n = 3$ we have $9 = 3^2$.
3. No, because $7 \in A$ but $7 \notin B$.
4. No, because $4 = 2^2 \in B$ but $4 \notin A$.
5. No, because as (a) and (b) show, $9 \in A \cap B$.

Exercise 6.3

These are all the points in the plane with integer coordinates, for whom the distance from the x-axis plus the distance from the y-axis is less or equal to 3. We can enumerate them from top to bottom:

$$\begin{array}{rl} y = 3: & (0,3) \\ y = 2: & (-1,2),(0,2),(1,2) \\ y = 1: & (-2,1),(-1,1),(0,1),(1,1),(2,1) \\ y = 0: & (-3,0),(-2,0),(-1,0),(0,0),(1,0),(2,0),(3,0) \\ y = -1: & (-2,-1),(-1,-1),(0,-1),(1,-1),(2,-1) \\ y = -2: & (-1,-2),(0,-2),(1,-2) \\ y = -3: & (0,-3) \end{array}$$

Together this makes 25 elements for the set K.

Exercise 6.4

The elements of B are those subsets of A which have more than two elements, that is, 3 or 4 elements.
So: $B = \{\{a,b,c\},\ \{a,b,d\},\ \{a,c,d\},\ \{b,c,d\},\ \{a,b,c,d\}\}$.

Exercise 6.5

The formula on the right hand side counts every element of A and of B, so it counts every element of $A \cup B$ but counts the elements of $A \cap B$ twice, once as members of A and once as members of B. The formula on the left takes exactly care of that double counting by adding $|A \cap B|$.