

Exercise Sheet 6

Exercise 6.1

Let A be the set $\{x \in \mathbb{Z} \mid -6 < x < 6\}$ and consider the following subsets of A :

$$\begin{aligned} B &= \{-4, -2, 0, 2, 4\} \\ C &= \{0, 1, 2, 3, 4\} \\ D &= \{-1, 0, 1\} \end{aligned}$$

Compute the following:

- (a) $B \cup C$ (b) $B \cap C$ (c) $\overline{B} \cap D$
(d) $B \cap \overline{D}$ (e) $B \setminus D$ (f) $D \setminus B$

6 points

Exercise 6.2

Let $A = \{x \in \mathbb{N} \mid \exists n \in \mathbb{N}. x = 2n + 1\}$ and $B = \{x \in \mathbb{N} \mid \exists n \in \mathbb{N}. x = n^2\}$. Which of the following statements are true, which are false? Give (short!) justifications for your answers

- (a) $9 \in A$
(b) $9 \in B$
(c) $A \subseteq B$
(d) $B \subseteq A$
(e) $A \cap B = \emptyset$

1 point

1 point

1 point

1 point

1 point

Exercise 6.3

How many elements are there in the set $K = \{(x, y) \in \mathbb{Z}^2 \mid |x| + |y| \leq 3\}$? (A picture may help.)

2 points

Exercise 6.4

Let A be the set $\{a, b, c, d\}$. List all the elements of the set $B = \{X \in \mathcal{P}A \mid |X| > 2\}$.

2 points

Exercise 6.5

Let A and B be subsets of a set X which has finitely many elements. Prove the following formula about the cardinality of the sets A , B , $A \cup B$ and $A \cap B$:

$$|A \cup B| + |A \cap B| = |A| + |B|$$

2 points

Total points: 17

Stretcher Exercise 6

(You can earn two *bonus points* by answering this question. Send your solution via email directly to O.K.Klinke@cs.bham.ac.uk.)

Construct a one-to-one correspondence that shows that the two sets

$$A = [0, 1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\} \quad \text{and} \quad B = (0, 1] = \{x \in \mathbb{R} \mid 0 < x \leq 1\}$$

are of equal cardinality.