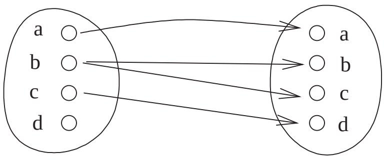
The University of Birmingham
School of Computer Science

Solutions to Exercise Sheet 8

Exercise 8.1

(a) 

(b) It is not, because it is not single valued: There are two outputs assigned to input b: b and c.

(c) It is, because for every possible input there is exactly one output.

(d) We change the second entry of R from (b,b) to (d,b). Now for every input there is exactly one output, but furthermore, different inputs are mapped to different outputs (injectivity) and all possible outputs do indeed occur (surjectivity), and these two properties together say that the function is a bijection, or in other words, a permutation.

Exercise 8.2

(a) $f(x) = x+1$ does the trick. It is not surjective because zero never appears as an output. (There are many other possibilities, for example, $f(x) = 2x$.)

(b) We can do the reverse of what is going on in (a): We map every $x \geq 1$ to $x-1$ and 0 to 0. The 0 appears twice as the output of both $x = 0$ and $x = 1$.

(c) There are functions in mathematics with the required properties: $\arctan(x)$ and $x^3 - x$, for example, but we can manufacture simpler examples by case distinction:

$$f(x) = \begin{cases} x+1 & \text{if } x \geq 0; \\ x-1 & \text{if } x < 0 \end{cases}$$

is injective but never outputs any of the numbers from the interval $[-1,1)$. Similarly,

$$g(x) = \begin{cases} x-1 & \text{if } x \geq 0; \\ x+1 & \text{if } x < 0 \end{cases}$$

produces every element in $[-1,1)$ twice.

Exercise 8.3

The problem is with the limited range of `int` which covers only the integers between -2^{31} and $2^{31}-1$. The method will only return the correct result if the input is between $\lfloor -\sqrt{2^{31}-1} \rfloor$ and $\lfloor \sqrt{2^{31}-1} \rfloor$. Using a calculator one finds the following for the allowable input range: $[-46340, 46340]$

One possibility is to work with `BigInteger` which is an implementation of arbitrarily large integers. Another is to insert a test and to throw an exception if the input is outside the interval $[-46340, 46340]$.

Exercise 8.4

(a) Since A is a 2×2 matrix and x is a 2×1 matrix, the result of the product Ax is again a 2×1 matrix. So it is clear that we have a function from V to V.

(b) Given a particular output matrix $y \in V$ we are looking for an input matrix x such that $y = Ax$. Multiplying from the left with the inverse matrix, this equations becomes $A^{-1}y = A^{-1}Ax = Ex = x$, in other words, the input matrix we are looking for is $A^{-1}y$.