Exercise Sheet 8

Exercise 8.1

Consider the set $A = \{a, b, c, d\}$ and the relation $R = \{(a,a), (b,b), (b,c), (c,d)\} \subseteq A \times A$.

(a) Draw a diagram of the situation in the style of Box 97. 1 point

(b) Is R a function from A to A? 1 point

(c) Is R^{-1} a function from A to A? 1 point

(d) Make one change to the definition of R so that it becomes a permutation on A. 1 point

Exercise 8.2

(a) Describe a function f from $\mathbb{N}$ to $\mathbb{N}$ which is injective but not surjective. 1 point

(b) Likewise, describe a function g from $\mathbb{N}$ to $\mathbb{N}$ which is surjective but not injective. 1 point

(c) Do the same for functions from $\mathbb{R}$ to $\mathbb{R}$. 2 points

Exercise 8.3

Consider the following Java method definition:

```
int square(int x) {return x*x;}
```

For which inputs `x` does it produce the desired output and for which does it produce nonsense? 2 points

How could the definition of `square` be changed to bring its behaviour closer to the mathematical squaring function? 1 point

Exercise 8.4

Let V be the set of 2×1 matrices. Let A be a fixed 2×2 matrix. Consider the operation that takes an element x of V and returns the result of the matrix multiplication Ax.

(a) Argue that this defines a function f from V to V. 1 point

(b) Argue that f is surjective if A is invertible. 1 point

Total points: 13

Stretcher Exercise 8

(You can earn two *bonus points* by answering this question. Send your solution via email directly to `O.K.Klinke@cs.bham.ac.uk`.)

Show the converse of question 8.4(b) above: If f is surjective, then A is invertible.
(Hint: You may want to re-read Item 50 on Handout 8.)