Solutions and mark sheet for first in-class test

Question 1

Compact notation:
$$\left(\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 8 \\ 3 & 5 & 0 & 8 \end{array}\right)$$
1 point

Eliminations in the first column:
$$\left(\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ 0 & -2 & 0 & 4 \\ 0 & 2 & -3 & -4 \end{array}\right)$$
2 points

Elimination in the second column:
$$\left(\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ 0 & -2 & 0 & 4 \\ 0 & 0 & -3 & 0 \end{array}\right)$$
1 point

Normalising the last two lines:
$$\left(\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$
1 point

Eliminations above the pivots:
$$\left(\begin{array}{rrr|r} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$
1 point

Stating the solution:
$$\begin{array}{rcr} x_1 & = & 6 \\ x_2 & = & -2 \\ x_3 & = & 0 \end{array}$$
1 point

Question 2

x_1 : freely chosen — 1 point

$$x_2 = \frac{1}{2}(2x_3 - x_4) = x_3 + 2$$
2 points

x_3 : freely chosen — 1 point

$x_4 = -4$ — 1 point

Question 3

(a) $\vec{n}_{E_1} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ — 1 point

$\vec{n}_{E_2} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ — 1 point

(b) Using the formula for the distance of a point from a plane one obtains:
$$\frac{d_1 - \langle \vec{n}_{E_1}, P \rangle}{|\vec{n}_{E_1}|} = \frac{1 - (1-2+1)}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$
1 point

and
$$\frac{d_2 - \langle \vec{n}_{E_2}, P \rangle}{|\vec{n}_{E_2}|} = \frac{0 - (2+1-1)}{\sqrt{6}} = -\frac{2}{\sqrt{6}}$$
1 point

(c) Using the formula for the nearest neighbour of a point on a plane one obtains:
$$P_1 = P + \frac{d_1 - \langle \vec{n}_{E_1}, P \rangle}{\langle \vec{n}_{E_1}, \vec{n}_{E_1} \rangle} \cdot \vec{n}_{E_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{6} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7/6 \\ 4/6 \\ 7/6 \end{pmatrix}$$
2 points

and $$P_2 = P + \frac{d_2 - \langle \vec{n}_{E_2}, P\rangle}{\langle \vec{n}_{E_2}, \vec{n}_{E_2}\rangle} \cdot \vec{n}_{E_2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \frac{2}{6} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \\ 4/3 \end{pmatrix}$$ 2 points

Question 4

(a) Camera images of a 3D scene produce a two-dimensional picture. If one has more than one camera then one can hope to reconstruct the 3D scene from the images. 1 point

(b) All that is required is to substitute the coordinates of the points into the equations that define the planes:

$$Q_1 \text{ in } E_1: \ 1 \times 2 - 2 \times 1 + 1 \times 1 = 1$$ 1 point

$$Q_2 \text{ in } E_2: \ 2 \times (-2) + 1 \times 4 - 1 \times 0 = 0$$ 1 point

(c) The insight is that the direction of the line is given by the normal to the plane. The rest is straightforward:

$$X = Q_1 + s \cdot \vec{n}_{E_1} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$ 1 point

$$X = Q_2 + s \cdot \vec{n}_{E_2} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$ 1 point

(d) Equating the two representations gives us three equations for the unknowns s and t:

$$\begin{aligned} 2+s &= -2+2t \\ 1-2s &= 4+t \\ 1+s &= -t \end{aligned}$$ 1 point

By adding the last two equations we find $2 - s = 4$ or $s = -2$. It follows from the last equation, for example, that $t = 1$. One can now check that these values for s and t satisfy *all* three equations, so there is indeed a solution (as lines in 3D normally do *not* intersect). From the value for s we get for the point of intersection

$$Q = Q_1 - 2 \cdot \vec{n}_{E_1} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -1 \end{pmatrix}$$ 1 point

Total points: 27