

First in-class test

Question 1

Translate the following system of linear equations into the compact notation and then solve it by Gaussian elimination:

$$\begin{array}{rclcl} x_1 & + & x_2 & + & x_3 & = & 4 \\ x_1 & - & x_2 & + & x_3 & = & 8 \\ 3x_1 & + & 5x_2 & & & = & 8 \end{array}$$

7 points

Question 2

After running Gaussian elimination, we obtained the following echelon form:

$$\left(\begin{array}{cccc|c} 0 & 2 & -2 & 1 & 0 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Describe the set of solutions.

5 points

Question 3

Consider the following two planes space, given in normal form:

$$E_1 : x_1 - 2x_2 + x_3 = 1 \quad \text{and} \quad E_2 : 2x_1 + x_2 - x_3 = 0$$

and the point $P = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

- (a) Write out the normals to E_1 and E_2 explicitly as vectors with three coordinates.
- (b) Compute the distance of P to both E_1 and E_2 .
- (c) Find the nearest neighbour to P on both E_1 and E_2 .

2 points

2 points

4 points

Question 4

(This is about the converse of previous question.) Consider the points $Q_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $Q_2 = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix}$.

We are trying to find the point Q for whom Q_1 is the nearest neighbour on E_1 and Q_2 is the nearest neighbour on E_2 .

- (a) Why is this a common problem in computer vision?
- (b) Show that Q_1 lies on E_1 and Q_2 on E_2 .
- (c) The points which have Q_1 as their nearest neighbour on E_1 form a line; give its parametric representation. Likewise for the line of points that have Q_2 as their nearest neighbour on E_2 .
- (d) Intersect the two lines to find Q .

1 point

2 points

2 points

2 points

Total points: 27

Formulas

Distance of a point P from the plane $\langle \vec{n}, X \rangle = d$: $\frac{d - \langle \vec{n}, P \rangle}{|\vec{n}|}$

Nearest neighbour P' of point P on the plane $\langle \vec{n}, X \rangle = d$: $P' = P + \frac{d - \langle \vec{n}, P \rangle}{\langle \vec{n}, \vec{n} \rangle} \cdot \vec{n}$