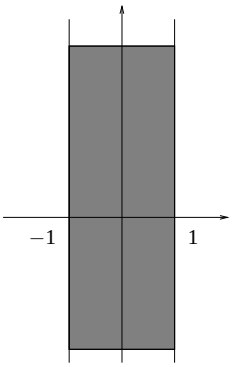
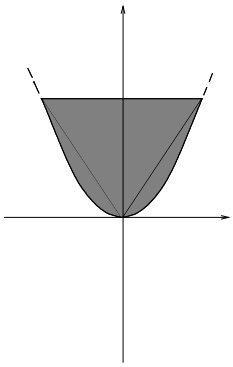
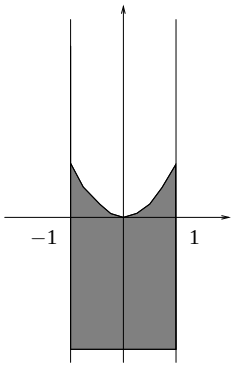
Solutions to the second in-class test

Question 1

(a) 

(b) 

(c) 

Question 2

(a) A_1 has six elements

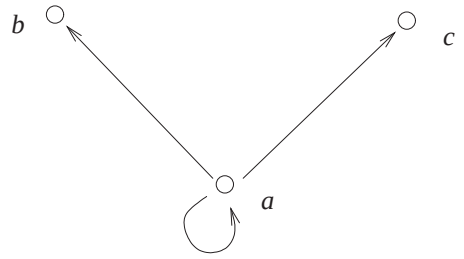
(b) A_2 is a countably infinite set; it consists of six copies of $\mathbb{N}$.

(c) A_3 is a countably infinite set

(d) A_4 is an uncountable set

(e) A_5 is an uncountable set

Question 3

(a) 

(b) (i) R is not reflexive because it lacks (b,b) and $(c.c)$.
(ii) R is not irreflexive because it contains (a,a).
(iii) R is not symmetric because it lacks (b,a) and (c,a).
(iv) R is antisymmetric because it does not contain (x,y) and (y,x) for two different points x and y.
(v) R is transitive because there are no paths to consider.

(c) We only need to make it reflexive, so we add (b,b) and (c,c).

(d)

$$\begin{aligned} \text{refl-closure}(R) &= R \cup \{(b,b),(c,c)\} \\ \text{symm-closure}(\text{refl-closure}(R)) &= \text{refl-closure}(R) \cup \{(b,a),(c,a)\} \\ \text{trans-closure}(\text{symm-closure}(\text{refl-closure}(R))) &= \text{symm-closure}(\text{refl-closure}(R)) \cup \{(b,c),(c,b)\} \end{aligned}$$

In fact, the generated equivalence relation is all of $A \times A$.

(e) R is not a function because it is not defined for b or c, and returns three possible values for a. R^{-1}, on the other hand, is a function; it maps all elements of A to a.

(f) R^{-1} is not injective, because more than one element is mapped to a. It also not surjective because neither b nor c appear in the output. Consequently, it is not bijective either.

Question 4

(a) In both Java and mathematics, one specifies the set from which the input is taken and the set in which the result will be. Java methods can be composed just like mathematical functions, and the formalism for this is the same as well. Like mathematical functions, a Java method returns a single result (if it returns anything at all).

(b) As for differences, a Java method may not return any result at all (and this could be a deliberate feature). Also, Java methods can have internal memory and hence return different answers at different times, even if the same input is the same.

Question 5

(a) If R and S are reflexive, then both contain all tuples (x,x) for any $x \in A$. Therefore, $R \cap S$ also contains these tuples.

(b) If $R \cap S$ contains the tuple (x,y) then this must be contained in both R and S. Because these are assumed to be symmetric, they both contain the tuple (y,x) as well. This is then also in $R \cap S$.

(c) If $R \cap S$ contains the tuples (x,y) and (y,z), then these must be contained in both R and S. Because R and S are assumed to be transitive, they both contain the tuple (x,z) as well. This is then also in $R \cap S$.

(d) The equivalence class of an element $x \in A$ with respect to $R \cap S$ contains all elements of A which are related to x by a link that is contained in both R and S. This means that such elements would be contained in the equivalence class of x with respect to R and also in the one with respect to S. The converse is also true. From this consideration it follows that the equivalence classes of $R \cap S$ are obtained by intersecting those of R with those of S. (Empty intersections are thrown away.)