Exercise Sheet 2 (5%) – Model Solutions

1. Pareto-optimal solutions are the *optimal* solutions *found* in a multi-objective optimization problem. Pareto-optimal front: the curve formed by joining *all* the Pareto-optimal solutions. Note: Optimal means non-improvable. So these will lie at the boundary of the feasible region.

2.a. The optimal Pareto front is indicated by thick grey lines on the figure below. To convince ourselves that this solution is correct, we additionally colour the parts of the search space dominated by the selected segments in yellow.

![Diagram showing Pareto-optimal front and search space domination]

2.b. The point $y_4$ divides the search space into four quadrants (visualised by dashed lines) in the above figure. For the given optimisation problem, all search points in the upper left quadrant (i. e., $x_4$) dominate $y_4$, all search points in the lower right quadrant are dominated by $y_4$ (i. e., $y_2$, $y_3$ and $y_5$). The remaining points are incomparable to $y_4$ (i. e., $x_1$, $x_2$, $x_3$, $x_5$, $y_1$).

2.c. NSGA-II performs non-dominated sorting on the parent and offspring population, i. e., on $P \cup Q$. For the given population we get 4 different fronts:
- $F_0 = \{x_4; x_5; y_1\}$ (blue)
- $F_1 = \{x_1; x_2; x_3; y_4\}$ (red)
- $F_2 = \{y_2; y_3\}$ (brown)
- $F_3 = \{y_5\}$ (black)

2.d. We have to select 5 individuals for the next iteration. Based on the above non-dominated fronts, we select all individuals in $F_0$ and discard all individuals in $F_2$ and $F_3$. So we are left with selecting the remaining two individuals from $F_1$. NSGA-II uses crowding distance sorting to decide, which individuals to choose. In the present case, $x_2$ and $x_3$ will be selected as these are the two `boundary' solution with crowding distance 1. We get: $P_{t+1} = \{x_2; x_3; x_4; x_5; y_1\}$. 