Shortcoming of Non-Elitist MOEAs

• Elite-preservation is missing

• Elite-preservation is important for proper convergence in SOEAs

• Same is true in MOEAs

• Three tasks
  – Elite preservation
  – Progress towards the Pareto-optimal front
  – Maintain diversity among solutions
Elitist MOEAs

Elite-preservation:
- Maintain an archive of non-dominated solutions

Progress towards Pareto-optimal front:
- Preferring non-dominated solutions

Maintaining spread of solutions:
- Clustering, niching, or grid-based competition for a place in the archive
Elitist MOEAs (cont.)

- Distance-based Pareto GA (DPGA) (Osyczka and Kundu, 1995)
- Thermodynamical GA (TDGA) (Kita et al., 1996)
- Strength Pareto EA (SPEA) (Zitzler and Thiele, 1998)
- Non-dominated sorting GA-II (NSGA-II) (Deb et al., 1999)
- Pareto-archived ES (PAES) (Knowles and Corne, 1999)
- Multi-objective Messy GA (MOMGA) (Veldhuizen and Lamont, 1999)
- Other methods: Pareto-converging GA, multi-objective micro-GA, elitist MOGA with coevolutionary sharing
Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

Non-dominated sorting: $O(MN^2)$

- Calculate $(n_i, S_i)$ for each solution $i$
- $n_i$: Number of solutions dominating $i$
- $S_i$: Set of solutions dominated by $i$
NSGA-II (cont.)

Elites are preserved
NSGA-II (cont.)

Diversity is maintained: $O(MN \log N)$

Overall Complexity: $O(MN^2)$
NSGA-II Simulation Results

NSGA-II (binary-coded)
Strength Pareto EA (SPEA)

- Stores non-dominated solutions externally
- Pareto-dominance to assign fitness
  - External members: Assign number of dominated solutions in population (smaller, better)
  - Population members: Assign sum of fitness of external dominating members (smaller, better)
- Tournament selection and recombination applied to combined current and elite populations
- A clustering technique to maintain diversity in updated external population, when size increases a limit
SPEA (cont.)

- Fitness assignment and clustering methods

In the context of SPEA, the population (Population) and external population (Ext_pop) are assigned fitness values based on their positions in the function space. Clustering methods, such as those based on distance (d) and peak maximum (p_max), are used to group solutions and determine their fitness assignments. The diagram illustrates how solutions are distributed across the function space and how clustering techniques can be applied to refine the fitness assignment process.
Pareto Archived ES (PAES)

- An (1+1)-ES
- Parent $p_t$ and child $c_t$ are compared with an external archive $A_t$
- If $c_t$ is dominated by $A_t$, $p_{t+1} = p_t$
- If $c_t$ dominates a member of $A_t$, delete it from $A_t$ and include $c_t$ in $A_t$ and $p_{t+1} = c_t$
- If $|A_t| < N$, include $c_t$ and $p_{t+1} = \text{winner}(p_t, c_t)$
- If $|A_t| = N$ and $c_t$ does not lie in highest count hypercube $H$, replace $c_t$ with a random solution from $H$ and $p_{t+1} = \text{winner}(p_t, c_t)$.

The winner is based on least number of solutions in the hypercube.
Niching in PAES-(1+1)
Constrained Handling

- Penalty function approach
  \[ F_m = f_m + R_m \Omega(\vec{g}). \]

- Explicit procedures to handle infeasible solutions
  - Jimenez’s approach
  - Ray-Tang-Seow’s approach

- Modified definition of domination
  - Fonseca and Fleming’s approach
  - Deb et al.’s approach
A solution \( i \) constrained-dominates a solution \( j \), if any is true:

1. Solution \( i \) is feasible and solution \( j \) is not.
2. Solutions \( i \) and \( j \) are both infeasible, but solution \( i \) has a smaller overall constraint violation.
3. Solutions \( i \) and \( j \) are feasible and solution \( i \) dominates solution \( j \).
Constrained NSGA-II Simulation Results
Applications of MOEAs

- Space-craft trajectory optimization
- Engineering component design
- Microwave absorber design
- Ground-water monitoring
- Extruder screw design
- Airline scheduling
- VLSI circuit design
- Other applications (refer Deb, 2001 and EMO-01 proceedings)
Conclusions

• Ideal multi-objective optimization is generic and pragmatic
• Evolutionary algorithms are ideal candidates
• Many efficient algorithms exist, more efficient ones are needed
• With some salient research studies, MOEAs will revolutionize the act of optimization
• EAs have a definite edge in multi-objective optimization and should become more useful in practice in coming years