Homogeneous and Heterogeneous Island Models for the Set Cover Problem

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1) Motivation

- **Set Cover** Given a set S with m elements and a set $\{C_i\}$ of n subsets of S each one having a cost c_i, the set cover problem requires to find a selection of subsets of S that cover the whole set and which have minimum cost.
- **GSEMO** multiobjective EA on a 2-objective function (cost, uncovered elements). From Friedrich et al. (2010) approximation ratio $O(\log m)$ in polynomial time. How to make it parallel?
- **Our contribution** Analysis of the runtime of two different parallel EAs, running on μ processors, to solve SetCover with the same approximation ratio as GSEMO. Comparison of different topologies and migration strategies.

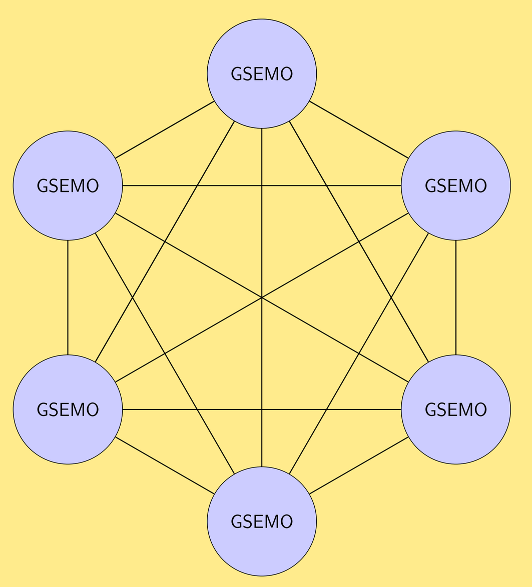
2) Measures

- **Parallel runtime**: the number of generations until at least one island has found a satisfactory solution
- **Communication effort**: the total number of individuals sent between islands, throughout a run of an island model.
- **Speedup**: the rate between the expected parallel runtime of the island model and the expected runtime of the same EA using only a single island.

If the speedup is of order $\Theta(\mu)$ we talk of **linear speedup**: it is the best possible use of parallelism we can hope for. For which μ can we guarantee linear speedup?

3) Homogeneus island model

μ islands



island population size $\leq m$

Each island: Runs GSEMO in parallel optimising the **same** 2-objective function and keeping non-dominated individuals to the next generation.

Sends copy of the population to all neighbouring islands with probability p.

Keeps the non-dominated individuals among its population, the offspring and all the immigrants.

2-objective fitness function: $f(X) = (u(X), \text{cost}(X))$

4) Runtime of the homogeneous model

From Lässig and Sudholt Theorem (2011):

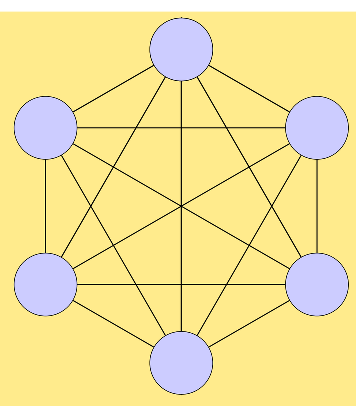
- Parallel runtime on the complete topology: $T_p = O\left(\frac{m}{p} + \frac{nm^2}{\mu}\right)$
- Communication effort (for any topology): $(m+1)pd\mu T_p$

  (where d is the degree of nodes and depends from the topology)

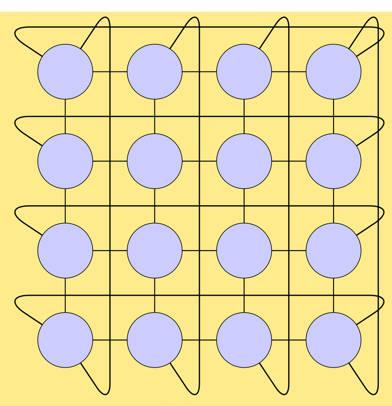
For the complete topology if $\mu \leq pnm$ the speedup is linear:

- Parallel runtime: $O\left(\frac{mn^2}{\mu}\right)$
- Best parallel running time: $O\left(\frac{m}{p}\right)$ (for $\mu = pnm$)

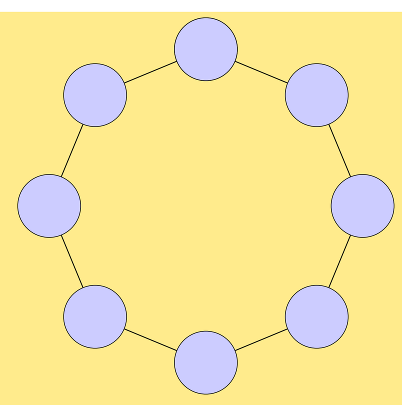
The degree of a node d, that affects the runtime, can be tuned changing the topology.



Complete



Torus

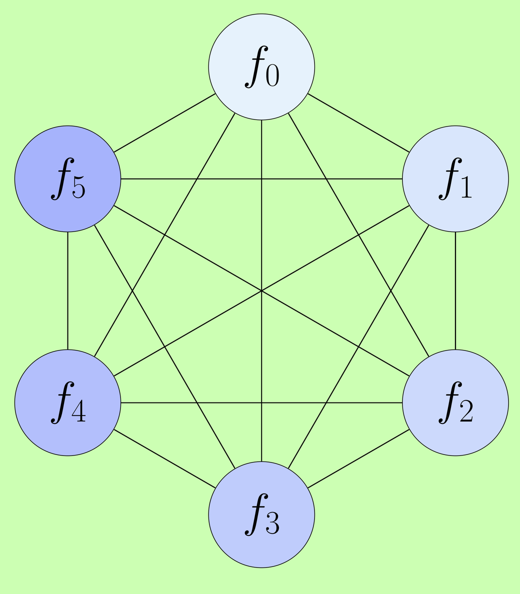


Ring

- Complete: $T_p = \left(\frac{m}{p} + \frac{nm^2}{\mu}\right)$, linear for $\mu \leq pnm$
- Torus: $T_p = \left(\frac{n^{1/3}m^{4/3}}{p^{2/3}} + \frac{nm^2}{\mu}\right)$, linear for $\mu \leq (pnm)^{2/3}$
- Ring: $T_p = \left(\frac{n^{1/2}m^{3/2}}{p^{1/2}} + \frac{nm^2}{\mu}\right)$, linear for $\mu \leq \sqrt{pnm}$

5) Heterogeneous island model

$m+1$ islands



island population size $= 1$

Each island runs a (1+1) EA on a **different** fitness function:

$$f_i(X) = \begin{cases} nc_{\max} - \text{cost}(X) & \text{if } c(X) = i \\ -|c(X) - i| & \text{if } c(X) \neq i \end{cases}$$

Island i represents the *keeper* of solutions covering i elements.

After migration each island keeps the best individual among its population, the offspring and all the immigrants.

$c(X) = m - u(X)$ is the number of covered elements, $c_{\max}$ is the maximum cost of a set.

6) Migration policies

The heterogeneous model requires a complete topology. So we study different migration policies running on the complete topology.

complete migration: each island sends migrants to all other islands.

uniform probabilistic: each island sends migrants to every other island independently with a migration probability p.

non-uniform probabilistic: each island i sends migrants to every other island $(i+k) \bmod (m+1)$ independently with probability $1/k$.

smart migration: each island i sends migrants to island $c(\tilde{X}_i)$, where $\tilde{X}_i$ is the offspring generated on island i.

7) Summary of results

(global) SEMO based homogeneous island models

Algorithm	parallel time bounds general b.	⇝ best bound	comm. effort
Non-parallel (global) SEMO	$O(nm^2)$	$\rightsquigarrow O(nm^2)$	0
– complete ($\mu \leq pnm$)	$O\left(\frac{nm^2}{\mu}\right)$	$\rightsquigarrow O\left(\frac{m}{p}\right)$	$O(p^2n^2m^4)$
– torus ($\mu \leq (pnm)^{2/3}$)	$O\left(\frac{nm^2}{\mu}\right)$	$\rightsquigarrow O\left(\frac{n^{1/3}m^{4/3}}{p^{2/3}}\right)$	$O(pnm^3)$
– ring ($\mu \leq \sqrt{pnm}$)	$O\left(\frac{nm^2}{\mu}\right)$	$\rightsquigarrow O\left(\frac{n^{1/2}m^{3/2}}{p^{1/2}}\right)$	$O(pnm^3)$

- There is an upper limit to the number of islands still leading to linear speed-up.
- Less dense topologies (i.e. torus, ring) assure linear speed-up only for a smaller maximum number of islands but lead to lower communication effort.
- The migration probability gives a smooth trade-off between the maximum number of islands that yields to linear speedup and the communication effort.

(1+1) EA (or RLS) based heterogeneous island models

$m+1$ islands running on μ processors to fairly compare it with the heterogeneous model

Algorithm	parallel time bounds general b.	⇝ best bound	comm. effort
– complete ($\mu \leq m$)	$O\left(\frac{nm^2}{\mu}\right)$	$\rightsquigarrow O(nm)$	$O(nm^3)$
– uniform prob. ($\mu \leq m$)	$O\left(\frac{nm^2}{\mu p}\right)$	$\rightsquigarrow O\left(\frac{nm^2}{p}\right)$	$O(nm^3)$
– non-uniform prob. ($\mu \leq m$)	$O\left(\frac{nm^2}{\mu}\right)$	$\rightsquigarrow O(nm)$	$O(nm^2 \log m)$
– smart migration ($\mu \leq m$)	$O\left(\frac{nm^2}{\mu}\right)$	$\rightsquigarrow O(nm)$	$O(nm^2)$

- A simpler model providing the same linear speed-up with less communication effort (with smart policy)
- Making each island investigating a different area of the search space might be a good idea

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