Introduction to quantum computing

Miriam Backens\(^1\) (they/them)
\texttt{m.backens@cs.bham.ac.uk}

School of Computer Science, University of Birmingham

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\(^1\)With thanks to Ashley Montanaro, whose slides parts of this talk are based on.
1. Introduction to quantum physics
2. What quantum computers are useful for
3. How to program a quantum computer
4. Building quantum computers
5. Conclusions
Outline

1. Introduction to quantum physics
2. What quantum computers are useful for
3. How to program a quantum computer
4. Building quantum computers
5. Conclusions
What is quantum physics?

The system of physical laws that govern very small things.

Pic: Wikipedia/Caffeine
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- Developed in **early 20th century** (and ongoing).
- Early applications include lasers, LEDs and transistors.
- There are many other quantum phenomena whose technological exploitation is only beginning.
Key properties of quantum mechanics

Very small things show behaviours that do not appear in the realm of everyday experience

1. Superposition: If a system can be in state A or state B, it can also be in a ‘mixture’ of the two.

2. Measurements: If we measure a system that is in a superposition of states A and B, we see either A or B probabilistically. Repeated measurements (without resetting) will yield the same result as the first measurement.

3. Uncertainty: There are pairs of measurements where greater certainty of the outcome of one measurement implies greater uncertainty of the outcome of the other measurement.

4. Entanglement: There exist states of multipartite systems which cannot be described in terms of states of the constituent systems.
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Superposition and measurement: Schrödinger’s cat

Pic: Wikipedia/Schrodingers_cat
Uncertainty (e.g. of position and momentum)

‘Do you know how fast you were going?’

‘No, but I know where I am.’

‘You were doing 90 miles an hour.’

‘Great, now I’m lost.’

Pic: anengineersaspect.blogspot.co.uk
The qubit: the basic building-block of a quantum computer

A quantum system with two distinct states is a **qubit**.

For example, a photon – a particle of light – has a property called polarisation which can be **vertical** or **horizontal** ($\uparrow$ or $\rightarrow$):

![Diagram of qubits: 0 and 1]
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![Diagram of qubit states: 0 and 1]

Just as a classical computer operates on bits, a quantum computer operates on qubits.
Entanglement

Imagine we have a pair of entangled qubits:


Even if we move one of the qubits to the Moon, the global state of the two qubits cannot be described solely in terms of the individual state of each of them!

In particular, if we measure one of the qubits, this apparently instantaneously affects the other one.

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Quantum simulation

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Simulating quantum physics has applications to drug design, materials science, high-energy physics, ...
1994: Peter Shor shows that quantum computers can factorise large integers efficiently.

Given an integer $N = p \times q$ for prime numbers $p$ and $q$, Shor’s algorithm outputs $p$ and $q$.

No efficient classical algorithm for this task is known.
**Shor’s algorithm for factoring**

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The quantum part of the algorithm uses **period-finding**: given a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ and the promise that there exists a number $a$ such that $f(x + a) = f(x)$ for all $x$, find $a$. 

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Shor’s algorithm breaks the RSA public-key cryptosystem on which Internet security is based.
Grover’s algorithm for unstructured search

Unstructured search is one of the most basic problems in computer science:

- Imagine we have \( n \) boxes, each containing a 0 or a 1. We can look inside a box at a cost of one query.

  
  0 0 1 0 0 0 1 0

- We want to find a box containing a 1. On a classical computer, this task could require \( n \) queries in the worst case.

1996: Lov Grover gives a quantum algorithm which solves this problem using about \( \sqrt{n} \) queries.

The square-root speedup of Grover’s algorithm finds many applications to search and optimisation problems, including in quantum machine learning.
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Pic: www.dcs.warwick.ac.uk/~tim/quantumcomputing/when/slide5.html
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The HHL algorithm for systems of linear equations

Solving a system of linear equations: Given a $N \times N$ matrix $A$ and a unit vector $b$, find the vector $x$ satisfying $Ax = b$. 

Aram Harrow, Avinatan Hassidim, Seth Lloyd (2008): Given $A$ and $b$, make a measurement on the quantum state described by the vector $x$ satisfying $Ax = b$. We don't get the solution $x$ itself. The matrix $A$ needs to be sparse. Running time is $O(\log(N) \kappa^2)$ vs $O(N \kappa)$ on a standard computer, where $\kappa$ is the 'condition number' of $A$ (roughly, the absolute value of the ratio between the biggest and smallest eigenvalue). Applications in science, engineering, machine learning and big data.
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![Picture](web.mit.edu/aram/www/)

![Picture](u.cs.biu.ac.il/~avinatan/)
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Secure quantum computing in the cloud

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The server learns nothing about the data or the type of computation.
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What quantum computations consist of

Measurements
- probabilistic
- irreversible
- lose ‘quantumness’

Unitary operations
- deterministic
- reversible
- keep ‘quantumness’

Unitary operations usually make up the bulk of a quantum computation. They are written down as quantum circuits:
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![Quantum circuit diagram with gates H, X, and Z]
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Unitary operations usually make up the bulk of a quantum computation. They are written down as quantum circuits:

Each horizontal wire represents a qubit, each gate represents an operation on one or more qubits.
Qubit states as vectors

A qubit state is described by a unit vector \( \begin{pmatrix} a \\ b \end{pmatrix} \) where \( a \) and \( b \) are complex numbers satisfying:

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|a|^2 + |b|^2 = 1
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Complex numbers matter: \( \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \) and \( \frac{1}{\sqrt{5}} \begin{pmatrix} -1 \\ 2 \end{pmatrix} \) give the same probabilities but they are different states.
A state of two qubits is described by a vector of length 4, whose components determine the probabilities of finding the two qubits in the states 00, 01, 10, and 11, respectively.

For example, \((1, 0, 0, 0)\) means both qubits are 0 and \((3/5, 0, 0, 2i/5)\) means either both qubits are 0 or both are 1 (this state is entangled).

A state of \(n\) qubits is described by a vector of length \(2^n\) whose components determine the probabilities for all the different \(n\)-bit strings.

For example, the three-qubit state \((0, 1/\sqrt{3}, 1/\sqrt{3}, 0, 1/\sqrt{3}, 0, 0)\) has equal probabilities of giving the bit strings 001, 010, or 100 when all qubits are measured.
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Reversible logic gates as unitary operations

The NOT gate \( \begin{array}{c}
X
\end{array} \) corresponds to the matrix \( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \):

\[
\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}
\]

i.e. \( \begin{align*}
0 & \rightarrow 1 \\
1 & \rightarrow 0
\end{align*} \)
Reversible logic gates as unitary operations

The **NOT gate** \( \overline{X} \) corresponds to the matrix \( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \):

\[
\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}
\]

\text{i.e. } \begin{align*}
0 & \mapsto 1 \\
1 & \mapsto 0
\end{align*}

The **controlled-NOT gate** \( \overline{\text{X}} \) corresponds to

\[
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ b \\ d \\ c \end{pmatrix}
\]

\text{i.e. } \begin{align*}
00 & \mapsto 00 \\
01 & \mapsto 01 \\
10 & \mapsto 11 \\
11 & \mapsto 10
\end{align*}
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The **NOT gate** \( \overline{X} \) corresponds to the matrix \( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \):

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\[i.e.\begin{cases} 0 \mapsto 1 \\ 1 \mapsto 0 \end{cases}\]

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\[i.e.\begin{cases} 00 \mapsto 00 \\ 01 \mapsto 01 \\ 10 \mapsto 11 \\ 11 \mapsto 10 \end{cases}\]

This is a reversible version of **XOR**, acting on bits as \( (x, y) \mapsto (x, y \oplus x) \).
Quantum gates with no classical counterpart

The Pauli-Z gate $\begin{array}{c}
\text{Z}
\end{array}$ corresponds to the matrix $\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}$:

$$\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}\begin{pmatrix}
a \\
b
\end{pmatrix} = \begin{pmatrix}
a \\
-b
\end{pmatrix}$$
Quantum gates with no classical counterpart

The Pauli-Z gate \( Z \) corresponds to the matrix \( \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \):

\[
\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ -b \end{pmatrix}
\]

The Hadamard gate \( H \) corresponds to \( \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \):

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \left( a + b \right)
\]

\[
= \frac{1}{\sqrt{2}} \left( a - b \right)
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The Pauli-Z gate \( \overset{\text{Z}}{\Box} \) corresponds to the matrix \( \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \):

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\]
Combining gates into circuits

Connect gates by (arbitrarily long) wires:
Combining gates into circuits

Connect gates by (arbitrarily long) wires:

Besides the gates introduced on the previous slides, there are many other gates that are commonly used in quantum circuits in different combinations.
Translating circuits to matrices

Two gates on the same wire correspond to the matrix product:

\[
\begin{pmatrix}
Z & H \\
\end{pmatrix}
\text{ is } \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}
\]

- Careful about the reversed order!
Translating circuits to matrices

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\[
\begin{pmatrix} Z \\ H \end{pmatrix} \quad \text{is} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}
\]

- Careful about the reversed order!

Two gates on parallel wires correspond to the Kronecker product (also called tensor product):

\[
\begin{pmatrix} H \\ Z \end{pmatrix} \quad \text{is} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}
\]

- This is not commutative.
Universality

The basic gates $H$, and $RZ, \theta$, corresponding to the matrices

$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}, \quad \text{and} \quad \begin{pmatrix}
1 & 0 \\
0 & e^{i\theta}
\end{pmatrix},
$$

are enough to write down a circuit for any unitary operation on a quantum computer.

Here, $\theta$ is an arbitrary real number, making $e^{i\theta}$ a complex number of absolute value 1.
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Some approaches to quantum computing

Photonics, Bristol

Ion trap, Oxford

Superconducting electronics, UCSB
Building a large-scale quantum computer is extremely challenging because of **decoherence**.

If a quantum computer interacts with the outside world and is subject to **noise**, it can lose its ‘quantumness’ and behave like a classical computer.
Quantum error correction

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- **Quantum error-correcting codes** can be used to fight decoherence.

Pic: DOI:10.1126/science.1253742
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If a quantum computer interacts with the outside world and is subject to **noise**, it can lose its ‘quantumness’ and behave like a classical computer.

- **Quantum error-correcting codes** can be used to fight decoherence.
- Optimistic estimates say error rates of up to 1% should be ok.
- Error-correction will **massively increase the number of physical qubits** needed to implement a given computation (by a factor of 1,000 or more).
Noisy Intermediate-Scale Quantum Computation

Often abbreviated to **NISQ**.
- Noisy: does not use error correction.
- Intermediate-scale: about 50-100 qubits.

Computations are **kept short** to avoid errors accumulating, but are expected to **outperform standard computers** on certain tasks.

Pic: WP/John Preskill

October 2019: Google announces they have performed a computation in 600 seconds on their chip of 53 superconducting 'transmon' qubits, which would take 10,000 years on standard computers, or 2.5 days on IBM's Oak Ridge Summit Supercomputer.
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- **Intermediate-scale**: about 50-100 qubits.

Computations are **kept short** to avoid errors accumulating, but are expected to **outperform standard computers** on certain tasks.

**October 2019**: Google announces they have performed a computation in **600 seconds** on their chip of 53 superconducting ‘transmon’ qubits, which would take **10,000 years** on standard computers, or **2.5 days** on IBM’s Oak Ridge Summit Supercomputer.
1. Introduction to quantum physics
2. What quantum computers are useful for
3. How to program a quantum computer
4. Building quantum computers
5. Conclusions
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Theory and implementation of quantum computers for the NISQ era and beyond are being *actively developed*.

There are still many interesting *open questions* about the power and potential of quantum computing to be explored.
Further reading

- Quantum Computing Since Democritus
  Scott Aaronson
  http://www.scottaaronson.com/democritus/

- Introduction to Quantum Computing
  John Watrous
  https://cs.uwaterloo.ca/~watrous/LectureNotes.html

- Quantum Computer Science
  N. David Mermin, Cambridge University Press

- Quantum Computation and Quantum Information
  Michael Nielsen and Isaac Chuang, Cambridge University Press

- Why Google’s Quantum Supremacy Milestone Matters
  Scott Aaronson
  https://www.nytimes.com/2019/10/30/opinion/google-quantum-computer-sycamore.html