Relaying in Wireless Networks Modeled through Cooperative Game Theory

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Abstract—We analyze the capacity of wireless networks in the presence of cooperative relaying by using game theory instruments. Cooperation is approached as a cross-layer interaction between routing and medium access control; the latter is assumed to be based on memoryless time-division multiplexing, where the users have fixed probabilities to access the channel. We investigate the proper cooperation mechanism to be adopted by the users so that a gain is obtained by both those who have their transmission relayed to the final destination and also those who act as relays. Especially, we show how this gain can be directly related to a throughput improvement if the users follow specific access procedures. This incentive to cooperation works not only in an abstract information theoretic context, but also for a more direct personal advantage of the users. Numerical results are shown to confirm the validity of the proposed approach. The adopted methodology is useful both for modeling and performance analysis of communication links in relay networks, and for designing viable protocols which the users have incentives to follow.

I. INTRODUCTION

Relaying networks have been widely studied in information theory [1]. In particular, the relay channel represents one of the most common scenarios studied. Several theoretical results about the capacity of this basic network have been available in the literature since long [2], and others keep being proposed even very recently [3], [4].

At the same time, game theory [5] is being employed more and more every day by wireless telecommunication engineers. From a game theoretic perspective, the relay channel is a natural scenario to evaluate cooperation [6], which may improve the communication for users experiencing bad quality on their direct link to the destination. The concept of cooperation has a precise meaning in game theory mostly through the application of coalitional games, which we will exploit in this paper.

Yet, we do not apply these game theoretic concepts to a pure information theory scenario. Instead, we focus on a precise application of the relay channel, with a real network protocol involving packet exchanges and retransmission, through cooperative Automatic Repeat reQuest (ARQ). Our proposal involves a cross-layer solution spanning on the data link (and more specifically, both channel access protocol design and ARQ) and network layers.

The contribution in this sense is two-fold. First, we give an analytical characterization of the performance of the relay channel from a game theoretic perspective. Second, we are also able to define a cooperative protocol where the overall network throughput is improved. However, differently from classic results of information theory where the capacity enhancement simply stems from spontaneous cooperation, i.e., the nodes collaborate out of goodwill, we precisely model also the node behavior so that their cooperation is not taken for granted, but rather promoted through a careful design of incentives to all the involved nodes (both those exploiting cooperative relays, and those who aid others, e.g., by forwarding their packets).

In this sense, this throughput enhancement is achievable in practical cases, and is truly beneficial for those nodes who have poor channel conditions, e.g., those at the cell edge, not only because somebody improves their throughput, but also since other nodes are actually willing to cooperate with them, since they see a concrete benefit in it.

The rest of this paper is organized as follows. In Section II we give some notions of coalitional game theory. In Section III we formulate our game-theoretic resource allocation strategy for a relay channel based on coalitional games and involving Medium Access Control (MAC), routing, and throughput subdivision. We present some numerical results in Section IV, then we sketch possible extensions in Section V and finally we conclude in Section VI.

II. COALITIONAL GAME TERMINOLOGY AND NOTATION

Cooperative game theory [6] is a branch of game theory that provides analytical tools to study the behavior of rational players when they cooperate.

The main area of cooperative games is represented by coalitional games [7], defined as a pair \((\mathcal{N}, v)\), where \(\mathcal{N} = \{1, ..., N\}\) is a discrete set of players and \(v\) is a function that quantifies the value of a coalition in a game. Each coalition \(S \subseteq \mathcal{N}\) behaves as a single player, competing against other coalitions in order to obtain a higher value of \(v\). A coalitional game may have the following properties:

Property 1. (Characteristic form) The value of a coalition \(S\) depends only on who are the members of that coalition, regardless of other coalitions

Property 2. (Transferable utility) The value of a coalition is a real number, representing the total utility achieved by the coalition, and it can be arbitrarily divided among its members

For coalitional games satisfying properties 1 and 2, the value \(v : 2^{\mathcal{N}} \rightarrow \mathbb{R}\) is a function that assigns to each coalition \(S\) the

This work has been supported by the EU project SAPHYRE, grant agreement no. 248001.
total utility achieved by it. The utility value can be arbitrarily divided among the coalition members and the amount of utility that a player \( i \in S \) receives, \( x_i \), is the player’s payoff. A payoff allocation is a vector \( x \in \mathbb{R}^{|S|} \) (where \(|S|\) is the cardinality of the set \( S \)) whose elements are the payoffs of players belonging to the coalition; in other words, it represents a redistribution of the total utility.

Another interesting property that a coalitional game may have is super-additivity, that for a game with properties 1 and 2 assumes the following form:

**Property 3.** (Super-additivity)
\[
v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \quad \forall S_1, S_2 \subset \mathcal{N} \quad s.t. \quad S_1 \cap S_2 = \emptyset
\]

The super-additivity property expresses in mathematical terms that formation of a larger coalition is always beneficial. Hence, for those games where it holds, the players are encouraged to stick together, forming the grand coalition \( \mathcal{N} \).

For a game having all properties listed before, the main aspects to analyze are:

- finding a redistribution of the total utility \( v(\mathcal{N}) \) such that the grand coalition is stable, i.e., no group of players has an incentive to leave the grand coalition
- finding fairness criteria for the redistribution of the total utility
- quantifying the gain that the grand coalition can obtain with respect to non cooperative behaviors

A payoff allocation is **group rational** if \( \sum_{i=1}^{N} x_i = v(\mathcal{N}) \) and it is **individually rational** if \( x_i \geq v(\{i\}) \) for every player that does not obtain a lower utility by cooperating than by acting alone. A payoff allocation having both properties is said to be an imputation.

The concept of core is also very important. It is defined as the set of imputations that guarantee that the grand coalition is stable, i.e., all payoff allocations where no group of players \( S \subset \mathcal{N} \) have an incentive to refuse the proposed payoff allocation, leaving the grand coalition and forming coalition \( S \) instead. Mathematically speaking,

\[
\mathcal{C} = \left\{ x \mid \sum_{i=1}^{N} x_i = v(\mathcal{N}), \sum_{i \in S} x_i \geq v(S) \forall S \subset \mathcal{N} \right\}
\] (1)

Indeed, the core may be empty, in which case the grand coalition is not stable. The existence of the core ought to be checked case by case, possibly exploiting some categories of games where the existence is guaranteed [5, Ch. 13].

### III. Problem Statement

We consider the scenario of two nodes, A and B, which want to communicate with an Access Point, Z, as represented in Fig. 1. \( \gamma_A, \gamma_B, \gamma_{AB} \) and \( \gamma_{BA} \) are the signal to noise ratios (SNRs) between A and Z, B and Z, A and B and B and A respectively. We suppose that:

- \( \gamma_A, \gamma_B, \gamma_{AB} \) and \( \gamma_{BA} \) are constant over time, i.e., time invariant channels and fixed transmission powers of A and B. Actually, also slow time-varying channels can be included in this analysis. Moreover, without losing generality, we suppose \( \gamma_B > \gamma_A \)
- node A and B always have packets to transmit to Z

- time division multiple access (TDMA) is adopted, assuming that the access point Z manages it in a centralized manner. The process that assigns a slot to a new packet is independent identically distributed (i.i.d.) with \( P_A \) and \( P_B = 1 - P_A \) the probabilities to assign the slot to node A or node B respectively
- we consider an ARQ retransmission scheme with at most 1 retransmission (maximum total number of transmission \( F = 2 \)). In subsection III-C we will see how the analysis can be generalized for multiple retransmissions
- we focus on the uplink connection from the users to the access point, therefore we neglect the traffic from Z to nodes A and B

Once a new packet for node A is scheduled, the non cooperative transmission process of this packet can be represented by the Markov Chain in Fig. 2. Absorbing states \( R_A \) and \( N_A \) represent the events that the packet is received or not received by Z. Other states represent the actual number of packet transmissions performed by user A, so the initial state is state \( 1_A \). We define \( q(\gamma) \) as the probability that a packet is correctly received when the SNR is \( \gamma \). This function depends on the modulation scheme used and on the packet length. We define \( P_{R_A}^{NC} \) as the probability to be absorbed in state \( R_A \) in the non cooperative case.

\[
P_{R_A}^{NC} = q(\gamma_A) + (1 - q(\gamma_A))q(\gamma_A)
\] (2)

We define \( N_{A}^{NC} \) as the average number of transmissions of the packet in the non cooperative case.

\[
N_{A}^{NC} = q(\gamma_A) + 2(1 - q(\gamma_A)) = 2 - q(\gamma_A)
\] (3)

The transmission of a packet in the non cooperative case, from the choice of the user to packet reception (or to the maximum number of transmissions), is represented in Fig. 3. Initial state \( I \) represents the selection of the user that can transmit the packet. Users A and B are selected with probabilities \( P_A \) and \( P_B \), respectively. Once either user is selected, the
transmission process evolves analogously to the Markov Chain shown in Fig. 2. When the packet is correctly received by Z or the maximum number of transmissions is reached (i.e., an absorbing state is entered), another new packet is considered, again starting from state 1: a new user is selected, a new packet is transmitted, and so on. Renewal theory [8] allows to study this kind of situations. The beginning of each renewal cycle constitutes a regenerative epoch of the Markov process. The asymptotic metrics of the network can be obtained by studying the average behavior of the Markov process. The asymptotic bit rate of each user is calculated by considering the average number of transmitted bits and dividing it by the average time to absorption:

\[ BR^N_C = \frac{P_AP^NC_{RA}}{N^NC} \frac{N_{bit}}{T_{pkt}} \]  

(4)

where \( N^NC = P_AN^NC_A + P_BN^NC_B \) is the average number of transmissions for packet, \( N_{bit} \) is the number of bits in a packet and \( T_{pkt} \) is the time needed for a single packet transmission.

Finally, the asymptotic bit rate of the network for the non-cooperative scenario is given by:

\[ BR^N_C = BR^N_C + BR^N_C \]  

(5)

A. Cooperative ARQ

Now the performance of the network is evaluated for the case where cooperation is active, by means of the coalitional game framework. Nodes can cooperate, helping other nodes to retransmit a packet not correctly received by the access point.

We assume that the game satisfies properties 1 and 2. Note that in the two-user case, the former property is automatically satisfied. However, the property still holds true even if the analysis is extended to a network with more than two users, since the TDMA approach guarantees that different coalitions do not interact: each coalition tries to obtain the maximum throughput by using the slots assigned exclusively to it. For what concerns property 2, the problem of the throughput redistribution is addressed in subsection III-B.

The value \( v(\cdot) \) of the coalitional game is the throughput obtained by each coalition. In a two-user case, three coalitions are possible: the two coalitions formed by the single users, and the coalition formed by both users, i.e., the grand coalition \( \mathcal{N} = \{ A, B \} \). The value of each coalition is:

\[ v(\{ A \}) = BR^N_C \quad v(\{ B \}) = BR^N_C \]

\[ v(\mathcal{N}) = BR^C = BR^C_A + BR^C_B \]  

(6)

where \( BR^C_A \) and \( BR^C_B \) are the respective asymptotic bit rates for user A and B in the cooperative scenario.

During a cooperative transmission, the packet transmitted by a node is heard by Z and the other user who actively cooperates. In our formulation, cooperation implies that, if a packet is not correctly received by Z, its retransmission is carried out by the user who has the better signal to noise ratio, provided that it received the packet correctly. Thus, the transmission process for user A can be represented by the Markov Chain in Fig. 4. State 2\_B (2\_A) represents the second transmission in the case that user \( B \) (user \( A \)) correctly received the packet during the first transmission; remember that we assume \( \gamma_B > \gamma_A \). We obtain:

\[ P^C_{RA} = q(\gamma_A) + (1-q(\gamma_A))(1-q(\gamma_{AB}))(1-q(\gamma_{BA}))(1-q(\gamma_{BA}))+q(\gamma_{AB})q(\gamma_B)) \]

\[ N^C_A = q(\gamma_A) + 2(1-q(\gamma_A)) = 2 - q(\gamma_A) \]  

(7)

Note that \( P^C_{RA} > P^NC_{RA} \) thanks to the cooperation of B, while \( N^C_A = N^NC_A \) because we are considering at most 2 transmissions. If more transmissions are considered, we obtain also \( N^C_A < N^NC_A \). Note that \( P^C_{RB} = P^NC_{RB} \) because B is not helped by anybody.

Along the same lines of (4), the asymptotic bit rate of user A in the cooperative scenario is given by:

\[ BR^C_A = \frac{P_A P^C_{RA} N_{bit}}{N^NC} \frac{1}{T_{pkt}} > BR^N_C \]  

(8)

Finally:

\[ v(\mathcal{N}) = BR^C = BR^C_A + BR^C_B > BR^N_C + BR^N_C \]  

(9)

Therefore the game satisfies also property 3.
B. Throughput subdivision

Now we want to find a payoff allocation that belongs to the core and is fair under certain parameters. Note that, for a super-additive two player game, the core is not empty and coincides with the set of imputations.

In the considered game, the set of imputations is given by:

$$x_A = BR_A^{NC} + (1 - w)(BR_A^C - BR_A^{NC})$$
$$x_B = BR_B^{NC} + w(BR_B^C - BR_B^{NC})$$ \hspace{1cm} (10)

Here a cooperation weight denoted as $w$ is introduced to determine the throughput share that each user gets. Note that $w$ is introduced to give a proper incentive to both users to cooperate. In fact, only user A, whose channel quality to Z is worse, can directly benefit from being helped by user B’s cooperative relaying. However, user B can get an incentive to cooperate if this results in a larger throughput share.

By varying the value of the cooperation weight $w$ in the interval $[0, 1]$, we can obtain all the imputations. It is immediate to see that $x_A + x_B = v(N)$ \forall w. Moreover, for $w = 0$ we obtain $x_B = BR_B^{NC} = v(\{B\})$ and increasing $w$ we have $x_B > v(\{B\})$. For $w = 1$ we obtain $x_A = BR_A^{NC} = v(\{A\})$ and decreasing $w$ we have $x_A > v(\{A\})$. On the other hand, if $w < 0$ then $x_B < v(\{B\})$ and if $w > 1$ then $x_B < v(\{A\})$. Thus, by setting the value of $w$ we decide the right level of fairness of the subdivision. This takes into account that, in order to cooperate with A, user B has to consume more power, transmitting packets instead of A, that can in turn save power. We can assign a cost to the power, depending on the application/scenario we are considering. If the cost of the power increases, we have to increase also the value of the cooperation weight $w$ (i.e., to further increase the payoff of cooperative users) in order to keep the same level of fairness.

So far we have supposed that the total throughput can be divided by users rather arbitrarily. From a practical point of view, the only thing that can be controlled is the allocation policy, $P_A$ and $P_B$. We suppose therefore that the allocation policy is changed from $P_A$ and $P_B$ to $P_A^C$ and $P_B^C$ in order to satisfy the subdivision proposed. Is the new allocation policy feasible? That is, is $P_A^C + P_B^C \leq 1$? It is easy to show that the new allocation policy is feasible. In fact, we have to increase the allocation probability of the cooperating user B and decreasing the allocation probability of A, while keeping constant the total bit rate $v(N)$. Since B has a better SNR, it results that the increase $P_B^C - P_B$ is greater that the decrease $P_A - P_A^C$ in order to keep the total bit rate constant. Therefore:

$$P_B^C - P_B < P_A - P_A^C \Rightarrow P_B^C + P_A^C < P_A + P_B = 1$$ \hspace{1cm} (11)

This means that the allocation is feasible and that there is a positive probability that some slots are not assigned to anybody, which would not be meaningful. Therefore, the quantity $P^C = 1 - P_A^C - P_B^C$ can be divided between users, increasing for example both $P_A^C$ and $P_B^C$ by the same amount, or increasing them by a weighted amount of $P^C$, where we can use again the cooperation weight $w$. Finally, this means that both users have a further benefit in obtaining an even higher bit rate compared to the subdivision proposed.

C. Multiple retransmission generalization

Previously, we have found the mathematical expressions for the asymptotic bit rates in the non-cooperative and cooperative cases for $F = 2$. This can be generalized to $F > 2$ as follows. For the non-cooperative case we obtain:

$$P_{RA}^{NC} = q(\gamma_A) + \sum_{i=2}^{F} q(\gamma_A)(1 - q(\gamma_A))^{i-1}$$

$$N_A^{NC} = q(\gamma_A) + \sum_{i=2}^{F-1} i q(\gamma_A)(1 - q(\gamma_A))^{i-1} + F(1 - q(\gamma_A))^{F-1}$$ \hspace{1cm} (12)

For the cooperative case, let $T_{A,Z}$ and $T_{A,B}$ be the times required by Z and B, respectively, to correctly receive the packet from A. Then, we have

$$P_{RA}^{C} = q(\gamma_A) + \sum_{i=2}^{F} P(T_{A,Z} = i)$$

$$= q(\gamma_A) + \sum_{i=2}^{F} \left[ \sum_{k=1}^{i-1} P(T_{A,B} = k) P(T_{A,Z} = i | T_{A,B} = k) + P(T_{A,B} > i - 1) P(T_{A,Z} = i | T_{A,B} > i - 1) \right]$$

$$= q(\gamma_A) + \sum_{i=2}^{F} \left[ \sum_{k=1}^{i-1} q(\gamma_{AB})(1 - q(\gamma_{AB}))^{k-1} q(\gamma_B) \right.$$ \linebreak
$$\cdot \left. (1 - q(\gamma_A))^k (1 - q(\gamma_B))^{i-k-1} + (1 - q(\gamma_{AB}))^{i-1} q(\gamma_A)(1 - q(\gamma_A))^{i-1} \right]$$ \hspace{1cm} (13)

$$N_A^{C} = q(\gamma_A) + \sum_{i=2}^{F} i P(T_{A,Z} = i) + FP(T_{A,Z} > F - 1)$$

$$= q(\gamma_A) + \sum_{i=2}^{F} \left[ \sum_{k=1}^{i-1} P(T_{A,B} = k) \right.$$ \linebreak
$$\cdot \left. P(T_{A,Z} = i | T_{A,B} = k) + P(T_{A,B} > i - 1) \right] + F \left[ \sum_{k=1}^{F-2} P(T_{A,B} = k) \right.$$ \linebreak
$$\cdot \left. P(T_{A,Z} > F - 1 | T_{A,B} = k) + P(T_{A,B} > F - 2) \right]$$ \linebreak
$$\cdot \left. P(T_{A,Z} > F - 1 | T_{A,B} > F - 2) \right]$$

$$= q(\gamma_A) + \sum_{i=2}^{F-1} \sum_{k=1}^{i-1} q(\gamma_{AB})(1 - q(\gamma_{AB}))^{k-1} q(\gamma_B) \right.$$ \linebreak
$$\cdot \left. (1 - q(\gamma_A))^k (1 - q(\gamma_B))^{i-k-1} + (1 - q(\gamma_{AB}))^{i-1} q(\gamma_A)(1 - q(\gamma_A))^{i-1} \right]$$ \linebreak
$$+ F \left[ \sum_{k=1}^{F-2} q(\gamma_{AB}) \right.$$ \linebreak
$$\cdot \left. (1 - q(\gamma_{AB}))^{k-1} (1 - q(\gamma_{AB}))^{F-k-1} + (1 - q(\gamma_{AB}))^{F-1} \right]$$ \hspace{1cm} (14)

It is easy to see that $P_{RA}^{C} > P_{RA}^{NC}$ and $N_A^{C} \leq N_A^{NC}$, therefore $BR_A^{C} > BR_A^{NC}$. Thus, the reasonings done in subsection III-B are still valid even for $F > 2$. 

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the case where assigned to the better user to encourage its cooperation. Thus, represent an improvement for this user either. In such a case, = 0 where both users get a cooperation gain, and have the proper (absolute) throughput gain. Its non-cooperative throughput. The larger B and the cooperative throughput of node A is higher than mine the actual benefit for the nodes in cooperating; therefore, when they decide to cooperate. Many related papers which they often resort to some form of side payment. However, point on the situation is reversed: when = 0 B which doubles its throughput with respect to the non-cooperative case. The closer B which doubles its throughput with respect to the non-cooperative case. The closer > γ, determines low throughput for the node with worse needs to give proper cooperation incentives so as to avoid the “ungrateful” situation (a user that most of the time enjoys cooperation by a relay does not reciprocate when it is its turn to relay a packet). Nevertheless, our preliminary evaluations hinted that a cooperation gain still holds also in this scenario. Finally, given the promising results found for a simple two-node network, it is surely worth investigating an extension to larger networks, possibly with multi-hop relaying. This development, currently under evaluation, implies both an evaluation on a larger scale and also the definition of a proper negotiation protocol to establish the cooperation roles.

IV. RESULTS

The throughput of the proposed scenario with a time invariant channel has been evaluated through a Matlab simulator. An example of results is shown in Fig. 5, where the throughput for both nodes in the cooperative and non-cooperative scenario is plotted. We consider a 16-QAM modulation with packet length of 12000 bits. PA and PB are equal to 0.5, γBA and γAB are both set to 110. The SNR of B, γB, is constant and equal to 70, while γA varies from 0 to 2γB = 140. For very low values of γA, in the non-cooperative case, the capacity of node A is close to 0, while that of B is around 2700 bits/s. In this case, most of the slots assigned to A are wasted to retransmit the same packet. When B cooperates, it allows to save most of this wasted slots without decreasing the throughput of A. This saved slots are then assigned to B which doubles its throughput with respect to the non-cooperative case. Increasing γA reflects in an enhanced non-cooperative throughput of A. Cooperation from B allows to save fewer slots than the previous case, but it is still beneficial and B experiences a higher throughput compared to the non-cooperative case. The closer γA to γB, the lower the gain of B. When the SNRs of both nodes are equal, cooperation is useless and the nodes have the same throughput. From this point on the situation is reversed: when γA > γB, A helps B and the cooperative throughput of node A is higher than its non-cooperative throughput. The larger γA, the higher the (absolute) throughput gain.

It is worth noting that, while not rewarding cooperation, i.e., w = 0, determines low throughput for the node with worse channel quality, cooperating with weight equal to 1 does not represent an improvement for this user either. In such a case, the whole throughput gain harvested from the cooperation is assigned to the better user to encourage its cooperation. Thus, the case where w = 0.5 represents an interesting trade-off, where both users get a cooperation gain, and have the proper incentives to cooperate from a game theoretic standpoint.

V. DISCUSSION AND POSSIBLE EXTENSIONS

The numerical results show an actual gain for both nodes when they decide to cooperate. Many related papers which discuss cooperation in wireless networks often fail to determine the actual benefit for the nodes in cooperating; therefore, they often resort to some form of side payment. However, the viability of such solutions, in both economic and legal terms, is arguable at best. Instead, our proposed approach simply addresses the benefit of cooperation as a throughput enhancement for both kinds of users, i.e., those relaying packets and those exploiting a relay. Also, focusing on a time-division multiplexing channel access is not restrictive; this assumption may easily be translated to other forms, e.g., random-based, medium access control.

Possible extensions of the present work, which are currently ongoing, involve the introduction of hybrid ARQ [9], as opposed to plain ARQ as employed here, and the addition of time-varying channels in the analysis. This latter point may require to extend the problem so as to include dynamic games and further negotiation among the cooperating users. In a time invariant channel, cooperation is always triggered in the same manner, i.e., the user with better channel relays the packets for the other; conversely, the main challenge of time varying gains would be that the users can sometimes have reversed cooperation roles. This complicates the set of strategies that can be played by each player; especially, the access point needs to give proper cooperation incentives so as to avoid the “ungrateful” situation (a user that most of the time enjoys cooperation by a relay does not reciprocate when it is its turn to relay a packet). Nevertheless, our preliminary evaluations hinted that a cooperation gain still holds also in this scenario.

Finally, given the promising results found for a simple two-node network, it is surely worth investigating an extension to larger networks, possibly with multi-hop relaying. This development, currently under evaluation, implies both an evaluation on a larger scale and also the definition of a proper negotiation protocol to establish the cooperation roles.

VI. CONCLUSIONS

We presented an analysis of a relay network by coalitional game theory, where a MAC/network protocol is designed by giving to all the users benefits when they collaborate with each other. This is concretely realized by proving the theoretical condition for a stable core and properly designing a suitable throughput subdivision among the users. The resulting solution properly accounts for modeling aspects such as users’ selfishness and thus the need for proper incentives to cooperation.

REFERENCES